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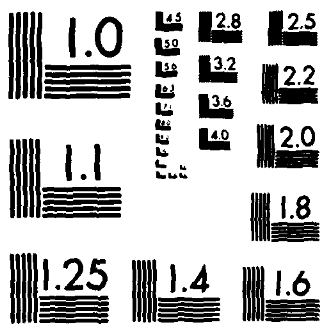

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A METHODOLOGY FOR IDENTIFYING COST

EFFECTIVE STRATEGIC FORCE MIXES

THESIS

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A METHODOLOGY FOR INDENTIFYING COST EFFECTIVE STRATEGIC FORCE MIXES

THESIS

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Captain, USAF

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Abstract

This thesis represents a methodology for the identification of cost effective strategic force mixes. The methodology makes use of Response Surface Methodology (RSM), economic production functions, economic theory, deterministic models, and Lagrangian techniques to identify cost effective choices. The methodology fits economic production functions to the response surface of a nuclear exchange model (a linear programming problem) using RSM. It then maximizes these economic production functions subject to a cost constraint using the Lagrangian technique. The use of economic production functions in this manner gives economic insight into the problem and results in the development of some simple buy decision rules for determination of cost effective force mixes. The classical use of polynomial models does not provide the same degree of information as the economic production functions, and information gained from the polynomial requires much more work.

A METHODOLOGY FOR IDENTIFYING COST EFFECTIVE STRATEGIC FORCE MIXES

I. INTRODUCTION

A. OVERVIEW

There has been a great deal of effort placed on quantifying military weapon decisions, the intent being to maximize some measure of merit, usually destruction. Oftentimes, cost is only included as an afterthought in these analyses, and not as a key ingredient in making proper optimal weapon buys. In this thesis, a nuclear exchange model which maximizes the damage expectancy of a number of different types of weapons will be analyzed. The analysis will make use of economics, Response Surface Methodology (RSM), and the properties of deterministic models to identify cost effective strategic force mixes. In essence, this combination of economics, RSM, and deterministic modelling represents a methodology for identifying cost effective choices with application to a broad range of problems.

The thesis explores the problem of identifying cost effective choices in a very general manner. The intent is that the results can be easily applied to other problems. The specific application of the methodology in this thesis is to a linear programming problem called a nuclear exchange

model, but any deterministic model could use RSM and economics to perform an analysis of cost-effective choices.

B. BACKGROUND

Response Surface Methodology (RSM) came into being as a theory in the early 1950's. Through the 50's and early 60's much work was done in developing RSM into a tool for studying the results of an experiment. Through RSM an experimenter could fit a postulated surface to his experimental results and thereby gain an understanding into possible mathematical relationships between the factors in his experiment. Additionally, optimal values of the surface could be located using various search techniques.

While RSM has been used extensively in experimentation, there has been little application to the results of deterministic models. In this context, a deterministic model will be any model that requires more than one functional relationship to determine the result. An example is a Linear Programming problem that has an objective function and one or more linear constraints. Deterministic models yield results quite different than experimentation. For example, there is no sampling error in the results of the deterministic model. That is, an experiment will yield slightly different results each time it is repeated, this is called sampling error, but a deterministic model will yield the same result every time it is applied. The output of a

deterministic model is the true surface in as much as the model conforms to the reality it is intended to simulate. Unfortunately, many deterministic models take an incredibly long time to calculate, even for a computer. The model may also represent so many combinations of factors, that it is not worthwhile to calculate the true value for each. RSM represents a method to "see" the resulting surface with a minimal of model runs.

Since the deterministic model is itself the true model of the phenomena, any postulated mathematical equation (also called a model) will not fit the surface perfectly and in effect will be biased. Bias represents systematic error and is error caused by an incorrect model being fitted to the data. A major defficiency currently in RSM is the lack of experimental designs that minimize this bias. An experimental design is an "optimal" set of factor values which minimize the number of observations required to minimize the variance error in fitting a functional form to the results. For instance, if one selects (postulates) a straight line as his model that would best fit his results (response), only two observations are needed to fit a line to the response. This represents the smallest design needed to fit the model. The theory of experimental design allows one to select the least number of points needed to accurately fit the model. Most of the designs used in RSM seek to minimize variance error and do not specifically take bias into account.

In fitting a model to a surface, Ordinary Least Squares (OLS) is the most common approach. The Sum of Squares of Error (SSE) is the total squared error between the postulated model and the actual surface. SSE is a combination of variance error and bias error. It is possible through the use of replications to find out what portion of the error is due to lack of fit (bias) or sampling (variance). In the application of models to the surface of a deterministic model, the SSE represents all bias error. Therefore, the model that most closely represents the true surface will be that which minimizes SSE.

Most mathematical models postulated to a reponse surface are first and second order models. While models of this form often give very good "fit" to the responses, the relationship among the coefficients of the various factors can be obscure due to magnitude differences in the inputs or difficulty in interpreting the meaning between linear coefficients, quadratic coefficients, and interaction coefficients of the same factor. The reason a second order polynomial gives such a good fit is that it represents "an approximating second order Taylor expansion of the true but unknown, response surface" (Box and Draper, 1971:734). An example of a second order polynomial is

$$Y = 2 + 6X + 5Y - 2XY + 3X^2 + Y^2.$$

There are other models which may lend themselves better for interpretation of the coefficients, but how good would they

graphically showing the effect of interactions on the response. Otto and Werner, 1983, used three-dimensional plots to show two-way interactions effect on certain chemical reactions (Otto and Werner, 1983:246, 263). Frazer and Brand, 1980, demonstrated the use of graphical techniques for data analysis using RSM.

Frazer and Brands' paper is an important divergence from the classic chemical analysis using RSM. Frazer attempts to fit the the response surface to a computer model of the surface. The abstract explains the intent of his paper. Frazer uses

> . . . a mathematical nonlinear least-squares technique that fits the entire experimental data set to the different kinetic models. We also show that when these NLSS mathematical techniques are used directly to test kinetic models, estimates of physical constants are improved, fewer graphical representations are required, and a most likely mechanism is more easily and dependably identified" (Frazer and Brandt, 1983: 1730).

Frazer is comparing the response surface of actual experimental data with the response surface generated by nonlinear models. Using these nonlinear deterministic models, he is able to more closely approximate the real underlying relationship. While there are some similarities to the thesis of this paper, Frazer's goal is quite different; he is trying to estimate an equation to a set of experimental data that contains measurement error.

In 1964, Wu authored a series of articles using RSM to predict tool wear. Whether it was a result of these arti-

analysis of variance test (ANOVA) when the model is simple, and the modified principal component analysis otherwise (Snee, 1972:61). The only article regarding analysis of response curves for deterministic models is Smith, 1979. Smith used RSM to perform a multi-dimensional parametric analysis of a model rather than the classical one dimensional sensitivity analysis.

C. APPLICATION OF RESPONSE SURFACE METHODOLOGY

RSM has been applied to numerous fields of science. Almost any three months of publication would yield a dozen applications. In this section three main applications will be discussed, the first as an example of the historical development of RSM, and the other two as they illustrate the broad range of RSM and potentially apply to this thesis subject.

RSM has found a great deal of acceptance in the chemical industry. Historically, this is where the first applications of RSM were employed. Today, there has been no abatement in the use of RSM in analyzing chemical experiments. A recent application is Otto, Rentsch, and Werner, 1983 ". . . optimal conditions for determination of CO(II) are deduced from response surface studies, considering the sensitivity and the blank absorbance as responses" (Otto and others, 1983:267). The chemical industry has used RSM for more than predicting responses, but also as a means of

be used to simulate a real life process, and provided the computer model does not employ any monte-carlo techniques, the result will always be the same. In other words, the computer has no measurement errors associated with its experiment, which means no variance. This requires the use of minimum bias designs. Very little work has been done in this area of applying RSM to deterministic models. Smith, 1979, is one exception. The payoff of applying RSM to deterministic models is great considering the tremendous run time of some computer models. To be able to predict the computer result through the use of some mathematical equation also provides insight into the problem.

While RSM is an important tool in the estimating of responses, to stop here would show a lack of appreciation for the full theory.

> It is of utmost importance that the experimenter be able to appreciate the results of an RSM study. This point cannot be overemphasized. There have been cases where rather elaborate mathematical models have been proposed, the appropriate data collected, the models fitted, and the results presented in the form of mathematical equations, only to have the practical implications of these results remain unappreciated and therefore unexploited (Hill and Hunter, 1966:573).

There are a number of good articles on the analysis of response surfaces. The three most referenced articles are Box, 1954; Box and Youle, 1955; and Snee, 1972. Snee reviews a couple of practical applications using principal component analysis, a modified principal component analysis, and a shape curve analysis. Snee recommends a general

as in controlling the environment of the experiment, instrument errors, etc.), while the bias error measures error caused by specifying an incorrect model to the surface. In most experimental design work, the focus has been on minimizing the variance error with little done to minimize the bias error. "Initially, criteria for judging goodness of designs were largely concerned with variance, ... The question of bias due to inadequacy of the . . . [model] was given somewhat secondary consideration" (Karson and others, 1969:461, 462). Very little work has been done in this area of developing designs to minimize bias error. Karson, Manson, and Haders' work in 1969, Karson, 1970, and Box and Draper, 1963, are the only articles located on this subject. Myers summarizes Box and Draper's work in one chapter in his book (Myers, 1976:196-218).

The reason that emphasis has been placed mainly on minimizing error due to variance is that most of the applications of RSM have been experiments. The most common early experiments were performed using different chemicals and environments to test reaction. In the development of RSM, it was very important to minimize the error due to measurement of the response in the laboratory.

Being able to minimize bias becomes a crucial issue when one moves from the application of RSM to experiments using imperfect measuring instruments to the application of computer generated responses to problems. The computer can

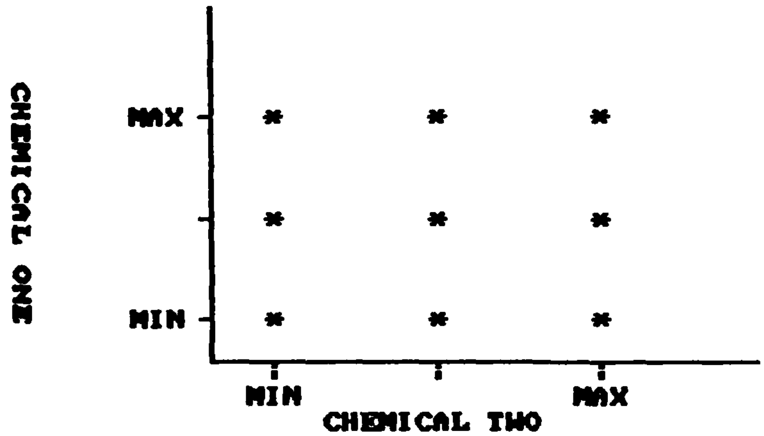

FIGURE 2.3

In analyzing the coefficients of a hypothesized model, it is very useful to have coefficients which are uncorrelated; in that way the error is not confounded. Confounded error means that it can not be *distinguished* which factors were the main contributors of the error, such as an interaction or quadratic term. In 1960 Box and Behnken suggested some possible new designs, "All the designs we discuss possess a high degree of othogonality; in fact, only the constant term . . . and the quadratic estimates . . . are correlated one with another" (Box and Bhenken, 1960:457). These designs, developed by Box and Bhenken, are employed in this study and are referred to as variance designs; that is, designs which seek to minimize variance.

The total error in a model can be divided into error due to variance error and error due to bias error. The variance error reflects measurement error of some kind (such

bined, enable the experimenter to make an efficient empirical exploration of the system in which he is interested (Myers, 1976:1).

Box and Wilson first developed the theory of response surface methodolgy in 1951. The theory continued to be expanded through the 50's and early 60's with the development of new experimental designs (Box and Hunter, 1957; Box and Draper, 1963). There are a number of different types of designs with varying degrees of usefulness, depending on the type model being fitted. The most common models are first-order and second-order. A first-order model is a model of linear form; a second-order model adds quadratic variables as well as two-way interactions. A first order model can also have interaction terms. The following equations illustrate these models.

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \qquad \text{(First-order model)}$$

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_{12} X_1 X_2 + \beta_{11} X_1^2 + \beta_{22} X_2^2 \qquad \text{(Second-order model)}$$

A design for a first-order model is called a first-order design and likewise, a second-order design for a second-order model. Myers, 1976, gives a good overview of the different type of designs.

An example of an experimental design is illustrated in figure 2.3. The minimum and maximum levels to be tested of the chemical are represented by MIN and MAX. The design is a simple uniform design measuring three different levels of each chemical for a total of nine combinations.

means that the model is biased. It will not fit the data perfectly due to this bias, the inability to completely specify the factors involved in the model. Variance error, the other type of error in an experiment, is caused by the inability to hold certain factors constant in the experiment and to perfectly measure the response. For both of these reasons, the model will usually not fit the responses as well as in figure 2.2. Using a technique such as least squares allows the determination of the error that is due to poor measurement of the responses or error due to an incorrect model postulated by the experimenter.

RSM helps in the design of the experiment in order to determine the least number of measurements needed to fit the model to the data. RSM defines the statistics used in calculating how accurate the postulated model is to the real data. If a mathematical model can be estimated using RSM to explain the process with a high degree of accuracy, then one no longer needs to perform the experiment to find out the response; responses are calculated using the mathematical model (for the combinations of factors within the limits of the experiment). Lastly, RSM uses techniques such as steepest ascent to find the combinations for optimal output.

> Response Surface Methodology . . . is essentially a particular set of mathematical and statistical methods used by researchers to aid in the solution of certain types of problems which are pertinent to scientific or engineering processes. . . . The response surface procedures are a collection involving experimental strategy, mathematical methods, and statistical inference which, when com-

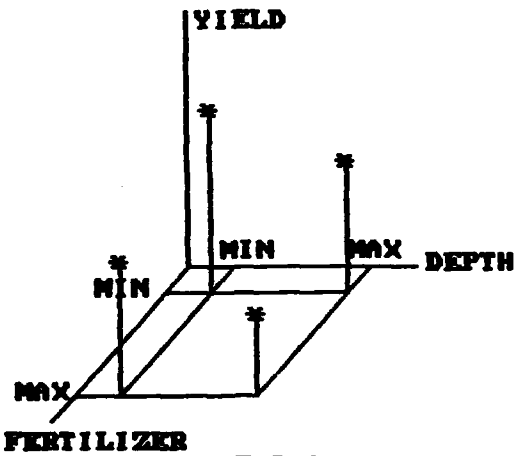

FIGURE 2.1

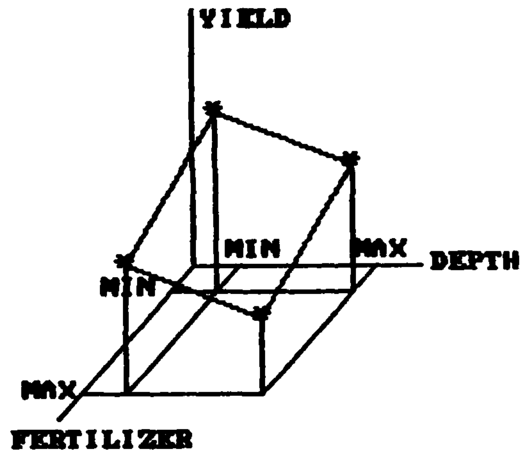

FIGURE 2.2

tried, and the response (number of bushels) is measured for each combination. Next, the scientist will attempt to fit a postulated mathematical model to the responses.

Figure 2.1 is a graphic picture of the example. Four different combinations of fertilizer amount and depth of planting have been employed. The response for each has been measured and plotted on the graph (the z axis represents yield, the x and y axis, fertilizer amount and depth). The responses, marked with an asterisk, represent the response surface. Using the method of least squares, a math model can be fitted to the response surface as in figure 2.2.

Obviously, there are many more factors involved in the yield of a crop then just fertilizer and depth of the cut. Other factors would be the amount of rain, amount of sun, type of soil, plant proximity to each other, weed influence, etc. The exclusion of these other factors in the model

II. LITERATURE REVIEW

A. INTRODUCTION

There is extensive literature on Response Surface Methodology (RSM), deterministic models, and production functions in economics, but little in applying all three together. The topic of this study, a methodology for identifying cost effective strategic force mixes, uses a combination of disciplines apparently unique to the scientific literature. Still, this combination of RSM, economics, and deterministic models suggests four potential areas in which to review the literature: Response Surface Methodology, application of RSM to deterministic models, economic applications using RSM, and economic production models.

B. THEORY OF RESPONSE SURFACE METHODOLGY

Response Surface Methodology is a method for fitting mathematical models to surfaces generated by experiment. These mathematical models are equations that the scientist has postulated will describe the response produced by his experiment. For example, suppose one wishes to know the yield of a particular crop depending on the amount of fertilizer and the depth of the seed planted. The response of the experiment, the yield of the crop, can be measured in number of bushels. Then, a number of different combinations of amount of ferilizer and depths of seed planted are

lowed by a three factor model. The following functions are estimated for the surface: 1) Polynomial, 2) Cobb-Douglas, 3) Generalized CES, 4) Multiplicative VES, 5) Additive VES. The results of each model are compared and the fit of the model tested. Also, relationships in the coefficients are examined; and the models evaulated to see what kind of interpretation can be given to each model.

Finally, a cost constraint is added to the deterministic model. Again, there are two approaches. One, the cost constraint can be added within the deterministic model. Two, it can be used outside of the model. The second approach would maximize the estimated function subject to a cost constraint using the Lagrangian method. The goal is to identify cost effective choices.

In summary, two major goals are to analyze the relationships among the different functions and to see how useful the additional economic interpretation is in giving the decision maker more information, specifically, information that helps in the determination of cost effective mixes.

tween the two types of models and economic interpretation of the results. Finally, add cost to these functions to see what type of cost effective results are given.

F. APPROACH

As a means of answering the questions that have been raised in this proposal, a deterministic model, with military aplications, will be used to test these ideas. The deterministic model is a nuclear exchange model that computes the optimal damage for a given number of weapons and weapon types. This is a common production problem with output measured in damage expectancy and the input variables are a given number of weapons per weapon type.

The first step is to research the nuclear exchange model to fully understand what exactly it is doing. What type of model is it? Does it contain any critical points?

Next, research is accomplished to find appropriate experimental designs with an emphasis on those which minimize bias. There is very little work done in this area, but it may be possible to use some present design which may have bias minimizing potential. Box and Draper, 1959, is a potential resource in this area. Additionally, a nonlinear estimation technique is required to fit the CES production function to the data. All other models are estimated using ordinary least squares.

After deciding on a design for each function, the computer runs are made. A two factor model is used, fol-

D. RESEARCH QUESTION

Can economic interpretations be obtained from the typical linear deterministic model and its response surface model? How does this differ from common experimental production functions? Can RSM and experimental design be used to fit production functions, such as the Cobb-Douglas, a generalized CES, a multiplicative VES, and an additive VES, to a response surface? How are these models interpreted and what relationships exist between the various models, including the second order polynomials? Which models fit the response surface best, and why is this so? What is the tradeoff between a functional form that predicts well and one that is easy to interpret and analyze? What happens when cost is added to the model, what type of answers will the response surface give? Does the use of economic production functions provide greater insight into determining cost effective mixes? How can others use this application in their analyses?

E. OBJECTIVES OF THE RESEARCH

The objectives of the research are to use RSM and appropriate experimental designs to apply common economic production functions to a response surface. Then evaluate these production functions with second order equations over a range of factor levels (number of variables in the equation). Compare the results and look for relationships be-

being introduced into the problem. If the SSE, which is all bias error, can be reduced to a sufficiently small percentage, then insight into the problem can be gained by studying the functional form, `ot to mention the savings from being able to predict results without having to rerun the deterministic model.

Finally, the most important interpretation of functions using RSM will be those involving cost. How are the results of the model modified by the addition of a cost restraint? There are two approaches in adding cost to the analysis. A cost restraint could be included in the deterministic model, or the estimated production functions could be maximized subject to a cost restraint. The question is then, where *does one include cost in the analysis*, i.e., as a restraint or as another function to subject the model to?

C. PROBLEM STATEMENT

It appears that little has been done to evaluate in economic (or other nonstatistical) terms the models used to capture the response surface of a deterministic model. What economic insights are revealed by response surface modelling and how do these compare with results of experimental models of similar economic phenomena? What information can be gained for identifying cost effective choices? A methodology is needed that evaluates and integrates RSM, deterministic modeling, and economics and into a form that will be useful to others using RSM in analysis.

fit the response? Also which is more important in a response surface equation, predicting ability or interpreting ability? Obviously these answers would depend on the application, but it appears to be an area that has had little research.

There are other functions, such as those used in economics, which could also be postulated as models. They are the Cobb-Douglas, the generalized Constant Elasticity of Substitution (CES), a Variable Elasticity of Substitution (VES) multiplicative model, and a VES additive model.

This last set of models, which have been used in economics, may have some advantage over a second order polynomial in that the coefficients have easily computed economic interpretations, such as marginal products, elasticities, etc. Other advantages, at least in the case of the Cobb-Douglas and CES production function are that they are not affected by magnitude differences among the input variables. The VES models contain interaction terms like a second order polynomial, and the functional expressions needed to calculate basic economic values are much simpler than the polynomial. Whether these models can predict as well as a second order polynomial is an interesting question.

No matter what the deterministic model is, the true surface very rarely will ever be captured perfectly by a mathematical equation. From the very start, bias error is

cles or not, the manufacturing industry has made wide application to various tool wear problems. Wu applies a couple of different models to approximate tool wear based on five input factors: speed, feed, depth of cut, angle, and nose radius. His articles are of special interest because he uses a multiplicative model to estimate tool life. This model is very similar in form to a Cobb-Douglas (C-D) production function, which is used in economics extensively and was used in this thesis. The Cobb-Douglas production function provides economic insight and interpretation to a process which would be much more difficult using an ordinary second order model.

Wu's use of a multiplicative function like the C-D was not to gain any economic insight, but rather because previous investigators had proposed a multiplicative functional relationship (Wu, 1964:106). Wu used RSM to estimate a multiplicative function for the purpose of saving time and money being spent on tool life measurement.

A third application of RSM is in nutrition. Two articles which discuss the use of RSM in feeding chicks are Toyomizu, Akiba, Horiguchi, and Matsumoto, 1982, and Roush, Petersen, and Arscott, 1979. These serve as excellent examples of the varied use of RSM, Toyomizu, et. al., used a creative triangular design to estimate a second order equation with three variables (Toyomizu and others, 1982:887). Both of these articles applied RSM to determining the

optimal combination of inputs to maximize the output.

There have been many other applications of RSM. These three were mentioned because they give an idea of the broad range of application of RSM.

D. ECONOMIC ANALYSIS USING RESPONSE SURFACE METHODOLOGY

The specific use of RSM in economics is scarce. Because of the nature of economics, specific applications would have to focus on developing large scale economic models, either deterministic or simulation, and then fit economic functions to these responses. Only in these applications would economics be fully utilizing RSM.

Burdick and Naylor discuss a potential application using Klein's six-equation econometric model of the United States and a Cobb-Douglas utility function (Burdick and Naylor, 1969:29-31). Unfortunately, they do not actually apply RSM; they only discuss how it could be used. This was the only article found on a specific deterministic application of RSM in economics.

Karlinger and Attanasi, 1980, use RSM to evaluate the effect of the risk of a flood on a person's willingness to buy flood insurance. They used a simulation program to model their problem and then, fit a utility function to the surface. This was the only other specific application of RSM in economics found.

The most likely reason for the lack of attention in economics to this area is that a model (deterministic or

simulation), is only an attempt to capture the real world, and the function fitted to the response surface of the model is an attempt to capture this surface. The potential for error is doubled with the use of two models.

> Now there are two response surfaces we can consider: one in the real world relating the real world response to real world factors and the other is the corresponding response surface in the model. It is our hope that the two will be very close, but this is much more likely to be true near the real world data points than far away from them. If our search leads us to an optimum point far from the real world data points, any resulting conclusions about the location of the optimum point in the real world should be made with great caution (Burdick and Naylor, 1969:31).

While economics has not made many specific applications of RSM, it has sought to capture data (real world data) in functional forms. Since the data are real world, it does not come in the experimental design form used in RSM. It is not experimental, but rather observation related. Similar to RSM, the techniques of least squares is widely used to fit models to real world data. These real world data are in a sense a response surface. The only difference is that it is random and not generated from an experimental design.

Therefore, a second approach in performing a literature review was to locate articles about well known production functions used in economics. This will form a nucleus of information that can be referenced later after attempting to fit these models to deterministic data.

There is a lot of literature concerning production functions in economics. Those with a good overall description include Ferguson, 1969; Henderson and Quandt, 1980; Nicholson, 1978; and Silberberg, 1978.

Zellner, 1966, deals specifically with the Cobb-Douglas (C-D) production function. The general form of the C-D for two variables is given in equation 2.1.

$$Y = \rho K^{\alpha} L^{\alpha-1} \tag{2.1}$$

For the purpose of explanation, consider the variables K and L to represent capital and labor. The powers of the variables K and L are the output elasticities of capital and labor. These measure the percent change in output (production) for a one percent change in an input (either capital or labor). As an example, if α equaled 0.5, then a one percent increase in capital would cause a 0.5 percent increase in production. The α and $1-\alpha$ terms are also a measure of the input intensities. The greater the α value, the more capital intensive the production process is (Brown, 1968:48).

The "ρ" value in equation 2.1 measures technical progress. Technical progress is an abstract concept capturing the importance of technology in the production process.

Arrow, Chenery, Minhas, and Solow first introduced a more general form of the C-D, a constant elasticity of substitution (CES) function, in 1961. Uzawa, 1962, generalized it for more than two parameters. Constant elasticity

of substitution means that the ability to substitute among the inputs and maintain constant output does not change for varying levels of the inputs. Equation 2.2 is an example of the CES for two inputs.

$$Y = \beta[\alpha K^{-P} + (1-\alpha)L^{-P}]^{-V/P} \qquad (2.2)$$

The variable β is an efficiency (or technology) parameter; α is a distribution parameter; P is a substitution parameter; and V measures returns to scale.

The technology parameter has the same abstract meaning as in the Cobb-Dougals above; it is a measure of the importance of technology in the production process.

The distribution parameter measures the intensity of each input in the technology. If α is large the technology is K intensive. In other words, the technology relies more on the input K than L to achieve its efficiency.

P measures the ease of substitution among the input variables (K and L). The ease of substitution is a difficult parameter to conceptualize. The higher the ease of substitution, the easier it is to substitute inputs and still maintain production, the lower the ease of substitution, the harder it is to substitute inputs for each other and maintain output. For high P values, which implies that the ease of substitution is difficult, the production process is referred to as proportional, that is, proportionate levels of each input are needed for production. For example one cannot trade labor for capital very easily in

this process. As a reference, the C-D's elasticity of substition is one, which is somewhere between low and high ease of substitution. Brown, 1968, is an excellent source for a detailed discussion of the CES as well as McFadden, 1963.

Both the C-D and CES are homogeneous production functions. A homogeneous production function is one where the relationship given in equation 2.3 holds; such a function is homogeneous of degree k.

$$f(tK, tL) = t^k f(K, L) \tag{2.3}$$

The most common degree of homogeneity is k = 1. In less technical terms, a doubling of both inputs would cause output to double. If a function is not homogeneous, a doubling of the inputs may not even have the same effect at different levels of the inputs. Two recent functions used in economics which are not homogeneous are the multiplicative nonhomogeneous (MNH) (Vinod, 1972:531-543) and the additive nonhomogeneous (ANH) (Sudit, 1973:499-514). Both functions are an attempt to further unrestrict economic problems for cases where variable elasticities of substitution (VES) are more appropriate. Vinod and Sudit applied their production functions to telecommunication problems where the ability to substitute among capital and labor changes over time.

Equation 2.4 is the MNH function. The 'A' is also a technology measure as in the C-D and CES. The elasticities

of output require calculation. The advantage of the MNH as a non-homogeneous production function is that most of the economic values can be easily calculated.

$$Y = AK^{\alpha+\mu LN(L)}L^{\beta} \tag{2.4}$$

The ANH does not have coefficients with simple interpretations, but like the MNH, the output elasticities, the marginal products, etc., can be easily computed.

Later in the thesis, the MNH and ANH will be explored more fully in a real example of the results.

The only article found relating economics and response surfaces is Burdick and Naylor, 1969. Their article is a good article for the economist attempting to understand the theory of response surface methodology. It does not give any specific examples of the application of RSM to economics, although it does suggest some applications.

E. CONCLUSION

Response Surface Methodology has been around for over thirty years and has had wide application in many fields. Most applications have dealt with experiments having some sampling error in the data. Very little literature has been written about the application of RSM to deterministic models. Deterministic models do not contain error due to sampling and therefore, require statistical designs that minimize bias error as opposed to error due to variance. Again, the literature on minimum bias designs is slight. This is to be expected given the history of application of

RSM to industry experiments. Additionally, little work has been done in the economic interpretation of RSM models.

The economic interpretation of RSM equations to deterministic models provides excellent further study. This work would develop a minimum bias design using the theory developed by Karson, the fitting of common economic production functions to deterministic data, and the comparison of the different models. In light of this literature review, this would represent the tying together of a number of theories to produce a useful document for the study of deterministic model responses.

III. METHODOLOGY

A. INTRODUCTION

The methodology used to identify cost effective strategic force mixes is composed of six major steps. The steps are as follows:

1) Select the deterministic model. (In this case a nuclear exchange model).

2) Choose potential production functions.

3) Select experimental design(s).

4) Run model for experimental design(s).

5) Estimate coefficients of production functions.

6) Maximize the functions subject to a cost constraint, and analyze the results.

The rest of the chapter will discuss steps one through five in more detail. Chapters IV and V will discuss step six for the two and three variable cases.

B. DETERMINISTIC MODEL

The deterministic model used to generate data for this study is a linear programming (LP) model. This model is a strategic weapons model. The model's intent is to maximize the amount of damage against a target base given a set of differing weapons. The LP model at first glance is fairly simple. The development of the model though is quite complex and includes weapon characteristics (Circular Error

Probability, yield, reliability, and daily alert probability), target characteristics (hardness and vulnerability to particular kill mechanism), and number of weapons assigned to a target (one or two) to yield a damage expected result for each weapon, target, and number of weapons used combination. The model was developed by Bunnell and Takacs in 1983. A good description of the model is found in appendix A of Graney, 1984.

A simplified example will help to better understand this model. The example consists of two weapons and two types of targets. The variables W_A and W_B will represent the number of weapons of type A and B available, respectively. The variables N_S and N_H will be the number of targets of type S and type H, for example, soft and hard. Figure 3.1 shows the amount of damage expected (D_1, D_2, . . . , D_8) given a particular weapon, target, and number of weapons per target configuration.

	Weapon A		Weapon B	
Number of Weapons per target :	1	2	1	2
Target S:	D_1	D_2	D_5	D_6
Target H:	D_3	D_4	D_7	D_8

Figure 3.1

The LP model now consists of maximizing the total destruction subject to limitations on the number of weapons

and the number of targets. The LP problem follows on the next page. In equation (3.1), X_1 represents the amount of targets S attacked with weapon A using one weapon per target, X_2 represents the amount of targets S attacked with weapon A using two weapons per target, and so on. Equations (3.2) and (3.3) are the limit of targets S and H available. Equations (3.4) and (3.5) limit the amount of weapons available. In these last two equations the factor '2' appears before the variables X_2, X_4, X_6, and X_8 to account for the fact that a unit of these variables is equivalent to two weapons used.

Maximize

$$Z = D_1X_1+D_2X_2+D_3X_3+D_4X_4+D_5X_5+D_6X_6+D_7X_7+D_8X_8 \tag{3.1}$$

subject to

$$X_1 + X_2 + X_5 + X_6 \leq N_S \tag{3.2}$$

$$+X_3 + X_4 + X_7 + X_8 \leq N_H \tag{3.3}$$

$$X_1 + 2X_2 + X_3 + 2X_4 \leq W_A \tag{3.4}$$

$$+X_5 + 2X_6 + X_7 + 2X_8 \leq W_B \tag{3.5}$$

$$X_1, X_2, \ldots, X_8 \geq 0$$

Admittedly, the model has some simplifying assumptions which do not fully capture reality. Theoretically, a more complex model would have also worked for the purpose of this study. Unfortunately, this was the only unclassified model available. Still, the general concern of this study is to examine production functions fitted to the response surface

of a deterministic model. Therefore, any LP model would work. The fact that our model has some potential military application is a bonus in this study.

The actual model developed by Bunnell and Takacs consisted of ten target types and five weapon types (Minuteman II, Minuteman III, Peacemaker (MX), B-1 bomber, and a submarine launched ballistic missile). In order to look for potential relationships or differences among a set of production functions, the model is reduced in size. Two cases are considered, one with two weapon types and the second with three weapon types. The number of targets (not the number of target types) are also reduced, sixty percent in the two variable case and fifty percent in the three variable case. The fifty and sixty percent reductions are purely arbitrary. They were meant to reduce the target base, but not so much that the weapons could exhaust the targets within the feasible range of weapons considered. In other words, rather than subject two or three weapons to a target base normally attacked with five weapons, a more realistic assumption is to reduce the target base and assume the other weapons are assigned to the rest of the target base. Another option is to leave the other weapons in the model at some constant value, but this causes difficult problems in fitting the production functions to the data. For example, the weapons set at a constant value do not produce the same effect for the varying levels of the other

weapons; and, the LP model chooses different targets for the weapons set at constant level. If this study had truly been to study the problem of nuclear exchange with the use of classified data, all weapons and targets would be left in the model, but since its purpose is to develop a methodology for examining such decisions with the use of other production functions, none of the actions to the model are of significance. The data in the model are not classified, so reducing the target base does not affect the use of the model. If the data were not reduced, the excess targets cause the LP model to place all the weapons on one or two targets, producing a linear effect in the results. Reducing the number of targets produces more interaction in the model between the weapons and the targets, providing a problem suitable for this research.

C. PRODUCTION FUNCTIONS

The functions or models used in the study to fit the data are classified into two main groups; those characterized by constant elasticity of substitution (CES) and those with variable elasticity of substitution (VES). These functions have all seen application in problems of production in economics, some much more than others. The Cobb-Douglas, a CES function, is seen widely in economic literature. On the other hand, the basic polynomial (of order greater than or equal to two), a VES function, is not seen very often in economic literature but appears commonly in other litera-

ture, such as operations research, as a function for fitting data. One purpose of this study is to examine the "fitting" ability of these various functions and compare that with the economic information gained from interpreting the coefficients.

The two types of CES functions used in this analysis are the Cobb-Douglas and the Arrow, Chenery, Minhas, and Solow design. These are referred to as the Cobb-Douglas (C-D) and the CES, respectively. The Arrow, Chenery, Minhas, and Solow function is called the CES function because it is the most general type of CES function.

The C-D assumes a production process that is somewhat "proportional"; that is, the inputs are not independent of each other. One can not have zero of one input and still have output. The CES allows zero of an input with output. In the LP model only one weapon is needed for output; therefore, in an aprior sense the C-D would not appear to be a good model for this problem, at least not when one of the inputs is near zero. The CES function does capture this essential aspect of the problem.

Four types of VES functions are used in the study, a linear model (LIN), a polynomial of degree two with interactions (POL), a multiplicative nonhomogeneous model (MNH), and an additive nonhomogeneous model (ANH). The MNH production function was developed by Koenker and Perry, and the ANH was developed by Sudit, both appeared in The Bell

the mean in economic terms. The variance design estimated polynomial is included because of its good fit (table 3.1).

MEAN ECONOMIC VALUES

	MARGINAL PRODUCT		OUTPUT ELASTICITY		RATE OF TECH. SUBSTITUTION OF W_2 FOR W_1		
	W_1	W_2	W_1	W_2	L	M	H
<u>BD Functions</u>							
Polynomial	.82	.76	.369	.566	0.94	1.09	1.34
CES	.82	.74	.371	.561	1.10	1.10	1.10
Linear	.82	.75	.375	.574	1.09	1.09	1.09
ANH	.85	.74	.390	.565	0.84	1.15	1.30
C-D	.78	.75	.333	.536	0.62	1.04	1.09
MNH	.32	.52	.136	.373	0.92	0.61	0.42
<u>VD Functions</u>							
Polynomial	.75	.69	.332	.536	0.92	1.09	1.53

Note: Mean values are Weapon 1 = 225 and Weapon 2 = 375. The last three columns represent a Low L: (50,50), Medium, M: (225,375), and High, H: (400,700), range of the inputs.

TABLE 4.2

The first four functions in table 4.2 agree quite closely on some basic economic measures. As a reminder, marginal product represents the marginal increase in output for a one unit increase in the input. The output elasticities measure the percent change in output for a one percent change in the input. The rate of technical substitution (RTS) of W_2 for W_1 is the number of units of W_2 which may be reduced when adding one unit of W_1 to maintain constant output. In economic terms the RTS is also the slope of the isoquant for a specified level of the inputs. An isoquant is the combinations of inputs that maintain a particular constant output. As can be seen from the range of the RTS

points better than the random data points. This is not unusual since the functions were estimated from the bias design data points and not the random data points. The polynomial is a remarkedly good fit of the points it was estimated from, and still an excellent fit to the random points. Two anomalities stand out though, the additive nonhomogeneous, while being a super fit to the bias design, is not nearly as good a fit to the random points. On the other hand, the CES is the third best fitting fuction to the bias design and the best fitting function to the random set.

In terms of the two measures of fitting ability (SDS and percent fit), the functions were subjectively broken into three major groups which could be called "marginal," "better," and "best." The "marginal" group includes the MNH and C-D. The "better" group contains the Linear and the ANH. The "best" group is the CES and the polynomial.

B. INTERPRETATION VALUE

Another method for determining which production function to use is to look at its economic interpretation. In the following sections of this chapter, each of these functions will be analyzed with respect to its economic interpretation and its subsequent implications. Before proceeding into the individual functions, it would be of interest to see how the functions compare to each other in economic terms. Table 4.2 compares the production functions at

functions fit extremely well. Unfortunately, it is not so clear which function fits best because the actual response is measured in neither of these extremes. The LP output ranged from zero to one thousand with the average response around five hundred. The SDS values in table 4.1 can be used to compare the functions against each other by examining the difference in the magnitudes of error between the function.

Another possible measure of fit is the percent fit of the predicted responses to the actual responses. Here, one must decide what is an acceptable percentage. If that value is defined as 95 percent, then three of the functions meet that criteria as well as one of the other functions estimated using the variance design.

Finally, it ought to be mentioned that the different measures of fit could give conflicting results. In which case the acceptance of a function may more depend on its interpretational value than on whether it is a slighty better fit than another function. It is interesting to note in table 4.1 that the percent fit of the polynomial is less than that of the ANH, yet the SDS's suggest that the polynomial would fit better.

Examining table 4.1, one sees that each of the six functions give slightly different conclusions when applied to the bias data base as opposed to the random data base. As one would expect, the functions fit the bias design data

fitting function. Since all the functions are estimated from the same design, the Sum of the Deviations Squared (SDS) of the bias design need only be compared. Table 4.1 compares the actual SDS of the functions estimated with the bias design when applied to the all bias design and the SDS when applied to the random data base. The last column in table 4.1 is the average for all the points of one minus the percent error (the predicted response minus the actual response, divided by the actual response).

TWO Variable Functions Estimated Using Bias Design

	****DATA POINTS****		PERCENT
	BIAS SDS	RANDOM SDS	FIT TO RANDOM
Linear	453.	1243.	.94
Polynomial	50.	555.	.96
Cobb-Douglas	4900.	26974.	.91
Mult. NonHom.	1242.	10305.	.88
Add. NonHom.	89.	1392.	.98
CES	349.	487.	.98

Note: BIAS design contains nine points.
Random design contains fifteen points.

TABLE 4.1

Before examining table 4.1, a couple of points need to be made. Alone, the SDS does not tell how good a fit the functions are to the surface. For instance, if the SDS is the only measure of fit and if the response is measured in units no greater than 10, the results of table 4.1 would suggest that none of the functions fit well. On the other hand, if the response is measured in billions, all the

IV. ANALYSIS OF THE TWO VARIABLE RESULTS

A. PREDICTION VALUE

There are two important elements in choosing a function to best represent the response of a surface. First, it should be able to accurately predict a response for a given set of inputs, and secondly, it ought to make sense in the interpretation of the coefficients, that is, either confirm an aprior theory or give insight that is not contradictory to the process generating the response. In the first part of this chapter, prediction will be the measure of effectiveness used in evaluating the production functions.

As was discussed in the previous chapter, in order to compare the various functions, a random set of inputs to the real surface was generated and input for each of the production functions. The random data set was not used to estimate any functions, but rather to compare the predicted values of the estimated functions against each other. The benefit of the random data set is that all the functions are compared against a common data set. Since some of the functions were estimated by different data sets (variance design or bias design), there was no common basis to compare the functions. The results (Table 3.1) showed that in all cases the bias design provided as good a fit or better to the surface than the variance design employed. Therefore, the functions with coefficients estimated with the bias design are the only functions to be considered for the best

E. ADDING COST TO THE PROBLEM

At the start of the study, some approaches were discussed for adding cost into the analysis. One suggestion was adding a cost variable in the function estimated using a response surface. In effect this would have meant adding another constraint, a cost constraint, to the LP model. Unfortunately, this does not work well since it produces quite conflicting results. The output, damage produced, really depends on the number of weapons or the total dollars. It doesn't depend on any combination of these two factors. Either there is sufficient funding and weapons restrain the production or there are sufficient weapons and funding restrains the problem. Inputting a funding level and weapon level into an equation does not tell which is the real driver in production (damage), and the response surface would tend to average the effect of the competing constraints.

A more realistic approach to adding cost to the problem is to maximize the particular production function subject to a cost constraint. This was done in the two variable case using a Lagrangian technique. For more than two variables, a nonlinear computer optimization program is really needed. The results of adding cost have quite different implications for the varying production functions and will be discussed more in the next chapter.

sign the all bias design would require seventy-six design points, the variance design, only fifty-four.

In dealing with the problem of multicollinearity in the all bias designs a couple of simple rules are used in the selection of variables to include in the model. If during the stepwise regression the adjusted R square decreases or the coefficients change drastically in sign, then the variable is not included and the process is stopped there. In cases where a coefficient was not estimated, it was usually the last variable left to be included in the regression.

The Box and Bhenken design used in this study requires the use of fifteen design points for the three variable case. These fifteen points include a center point replicated three times. For a deterministic model there is no need to replicate the center point since one will get the same response each time. An alternate idea to leaving the replications out is to use psuedo-replications, that is, moving off the center one unit in any direction. An advantage to doing this is that it provides more "weight" to the center of the design. Also, since the all bias design requires fifteen design points, it makes the two designs at least equal in number of design points. There is no noticable change in the orthogonality of the variance design by the use of psuedo replications.

The first drawback is the lack of orthogonality in the design for models of order two. The variance design possesses a high degree of orthogonality. Because of this fact and the lack of variance error in the data, all variables can be included when estimating using multiple regression. This is not always the case with the bias design. Because of the lack of orthogonality, there is a problem with multicollinearity. The coefficients can change drastically if too many variables are added to the model. A possible solution would be to use an orthogonal all bias design based somewhat on the Box-Bhenken designs, or on some other orthogonality technique.

Another benefit of orthogonal designs is that the error contribution of each term can be easily determined. This allows one to see which terms best fit the surface and which terms are most important in predicting. This can be accomplished because the variance/covariance matrix of an orthogonal design is zero everywhere but along the diagonal. The diagonal then represents the contribution of error from each term in the design.

The second drawback with the all bias design is the number of points to estimate. In the two cases in this study the number of designs points are the same, but in any design for more than four variables the all bias design would require the calculation of more data points than the Box-Bhenken designs. For example, for a five variable de-

TWO Variable Function Estimated By

	Variance Design		Bias Design	
	SDS		SDS	
PRODUCTION FUNCTION				
Linear	12466.	(.885)	1243.	(.942)
Polynomial	1676.	(.962)	555.	(.959)
Cobb-Douglas	45792.	(.823)	26974.	(.909)
Mult. NonHom.	240149.	(.546)	10305.	(.884)
Add. NonHom.	586907.	(.634)	1392.	(.980)
CES	+		392.	(.976)

THREE Variable Function Estimated By

	Variance design		Bias Design	
	SDS		SDS	
PRODUCTION FUNCTION				
Linear	87818.	(.897)	86875.	(.899)
Polynomial	6014.	(.981)	4187.	(.980)
Cobb-Douglas	1095204.	(.608)	228665.	(.876)
Mult. NonHom.	777413.	(.671)	128150.	(.909)
Add. NonHom.	+		+	
CES	+		45706.	(.932)

"+" - not estimated

Table 3.1

The analysis of the results of table 3.1 as they apply to the best function to use and its interpretation will be more fully discussed in Chapters IV and V. What is interesting to note here is the improvement in accuracy using a bias design against a more conventional variance design. It should not be forgotten that the data have no variance since it came from a deterministic model which implies that all the error is bias error. The MNH and ANH functions benefitted the most with the use of the bias design, at least in the two variable case. Unfortunately, there are some drawbacks in the use of an all bias designs to deterministic models.

Multiple regression is used to fit the experimental design responses to the production functions. The CES, as mentioned above, required the use of a nonlinear estimation technique.

Although it is not the specific intention of this study, a subgoal was to determine the practicality of using all bias designs to fit a function to data of a deterministic model. To measure the various fitting abilities of the functions, those estimated using a VD and those using a BD, a random set of data points to the real surface is generated and applied to each estimated function. The error is computed as the expected response minus the actual response. The Sum of the Deviations Squared (SDS) is compared for each of the functions estimated with different designs (VD or BD). In the application of variance designs and bias designs to fitting these functions it was discovered that the bias designs provide a better fit. Table 3.1 shows the result of this excursion using 15 randomly generated data points for the two variable case and the results of 25 random points for the three variable case. The number in the parenthesis could be called the "percent fit to the surface." It is one minus the average of the percent error. The percent error is the absolute value of the predicted response minus the actual response divided by the actual response.

All Bias Designs				
Variables:	1	2 . . .	k
	$\pm g$	$\pm g$ . . .	$\pm g$
	$\pm\alpha$	0 . . .	0
	0	$\pm\alpha$ . . .	0
	0	0 . . .	$\pm\alpha$
	0	0 . . .	0

Note: $g = ([ii])^{1/2}$ and $\alpha = 2^{k/4}g$ for rotatability where k = # of variables and $[ii] = 1/(k+2)$.

Figure 3.3

and the maximum value is set equal to positive one. The mean or middle value is then zero. The coded values can be used directly in the regression analysis or the actual variable values. The range used in this analysis is 0-450 for weapon one, 0-750 for weapon two, and 0-1040 for weapon three.

Some of the functions estimated are not second order. The C-D and linear function are first order functions. In the case of the ANH, it is not clear what type of design the function would require. The second order design is the best choice for a design since a second order design will estimate first order and second order models correctly. The CES model cannot be estimated using multiple regression and a nonlinear approach using the same experimental design was employed with the CES.

The first set of designs, the variance designs (VD), are a full factorial for the two variable case and a rotatable second order design developed by Box and Behnken for the three variable case (Box and Bhenken, 1960:455). Both designs possess a high degree of orthogonality. The designs for the two and three variable case are given in figure 3.2. The Box and Bhenken work contains designs for problems of up to sixteen variables.

DESIGN POINTS

Full Factorial (Two variable)		Rotatable Second-Order (Three variable)		
± 1	0	± 1	± 1	0
0	± 1	± 1	0	± 1
± 1	± 1	0	± 1	± 1
0	0	0	0	0

Figure 3.2

The second set of designs, all bias designs (BD), seek primarily to minimize bias. These designs are developed from the work of Box and Draper summarized in Myers' book on RSM (Box and Draper, 1959:622-654). They require the calculation of 'α' and 'g' from the second moment, generally noted in the literature as '[ii]'. The general design is given in figure 3.3.

In both figure 3.2 and 3.3, there is an assumption that coding is being used in the design. For these designs it is assumed that the mininum value is set equal to negative one

times 751) for the two variable case and 3.5258774E+08 runs (451 times 751 times 1041) for the three variable case. In the three variable case, with each run taking 0.40 CPU seconds, this would amount to four and one-half years of computer run time. Obviously the resource cost is too great to completely estimate the function by data points, and data points alone do not tell much about the relationships between the weapons. For this reason specific points are chosen and measured and functions are fitted to these points to estimate the true surface. Experimental designs provide the vehicle to estimate the surface with a mininum of data points.

Most designs used in fitting a function to the response attempt to minimize variance. In fitting a function to the response of a deterministic model the only error involved is bias error; there is no variance error. Bias measures the error in choosing the wrong function to fit the surface. In choosing an experimental design it seemed appropriate to use a design which minimized bias. Actually, Myers concluded from the work of Box and Draper that designs which minimize variance do a very good job at minimizing bias also (Myers, 1976:208, 230). Still, in this study two experimental designs are used, a variance minimizing design and a bias minimizing design. Both are second order designs, that is, designs specifically for fitting a second order equation to the data.

Journal of Economics and Management Science. The linear model is the simplest of the four, while the polynomial is the most complex. The polynomial model is used extensively in non-economic research. There is good reason for this wide use of the polynomial, because it is a quasi-Taylor expansion of the true and unknown function (Box and Draper, 1971:734).

Of these four functions the MNH would least likely fit the production process represented in the LP model. The MNH requires positive values of all inputs before there is any output. Still the MNH is chosen for use to see how it fits away from the origin and to see what kind of economic insight it gives.

All of these models have been discussed in more detail in chapter two of this report and further questions should be directed there or to the references in the bibliography.

D. EXPERIMENTAL DESIGN

The selection of an experimental design proved to be one of the more difficult aspects of this study. Since the data do not come from an actual experiment, but from a deterministic model, there is no variance error in the data. The data represent the actual or true surface. In fact if one had the computer resources he could calculate the response for every input combination. In the simple problem used in this study that would amount to 338,701 runs (451

values, the slope of the isoquants do not change much for differing levels of the inputs. These results also seem to suggest that the MNH is not a good model of the response surface. It's economic interpretation is completely out of line with the other functions which also fit the surface better. The Cobb-Douglas, while not agreeing with the other functions as well, is not that poor. Additionally, there is evidence to suggest the C-D's fit is fairly good away from the origin. This will be discussed more fully in the section on the Cobb-Douglas.

In choosing a best fit, these last two sections have examined both the predicting ability and a general economic interpretation at the mean. In order to more closely examine the economic interpretation, each function needs to be looked at in greater detail. The following sections will examine the functions beginning with the better fitting and ending with the least best fit.

1. <u>THE CES PRODUCTION FUNCTION</u>. The CES production function is a very common model in economic literature. Each of the parameters estimated in the CES function have very descript properties. The form of the CES is as follows:

$$Y = A(\alpha K^{-P} + (1-\alpha)L^{-P})^{-V/P}$$

A is an efficiency parameter $(A>0)$; P is a substitution parameter $(-1 \leq P < \text{infinity})$; α is a distribution parameter $(0 \leq \alpha \leq 1)$; V measures returns to scale $(V>0)$.

The CES function was estimated with the bias design using a nonlinear approach. Initially none of the values were restricted. Below is the result when the parameters are allowed to range freely.

$$Y = 2.23(.53W_1^{1.07} + .47W_2^{1.07})^{.95/1.07}$$

Surprisingly, one of the parameters when estimated did not fall in the prescribed range. The P parameter which measures the ability to substitute among the inputs was just barely less than negative one (-1.07). A P value of negative one indicates perfect ease of substitution. A large P value would indicate a proportional process. For example, mowing lawns is a proportional process with substitution extremely difficult. If there are three lawnmowers and three individuals to mow the lawns, one cannot substitute lawnmowers for more individuals to mow the lawns without losing some production. Perfect ease of substitution implies the opposite; it is very easy to substitute among the inputs and maintain production.

Perfect ease of substition is a seemingly obvious result for the case of two weapons producing damage. Therefore, P was set equal to negative one and estimated again by nonlinear estimation techniques.

$$Y = 2.34(.52W_1 + .48W_2)^{.94} \qquad (4.1)$$

One interpretation of equation 4.1 is that the process is slightly more weapon one intensive than weapon two, but what does this mean? In a more typical capital-labor pro-

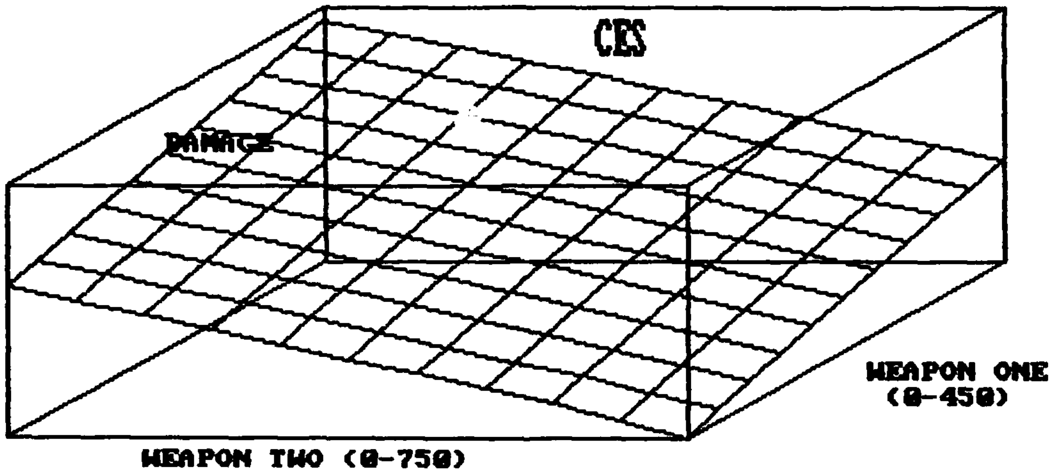

FIGURE 4.1

duction problem, if capital had a higher intensity value than labor, it means that the technology is more capital intensive. It appears that in this context, the intensities measure the capabilities of the weapons. The ratio of the intensities is the slope of the isoquant, for all isoquants. This will become clearer later in the discussion, hence, the ratio of the intensities is the rate of technical substitution. Therefore, the intensities reflect the mean capabilities of the weapons.

Also, equation 4.1 shows that the process has diminishing returns to scale (.94 < 1). There is proportionately less payoff for more weapons. This makes sense since as one increases the number of weapons, he must attack progressively more difficult targets. After the easier targets have been attacked there are only incremental increases in destruction for more weapons.

The efficiency parameter in equation 4.1 is a more difficult parameter to interpret unless there is another production process to compare it with.

Figure 4.1 is a plot of equation 4.1. The X-axis (Weapon 1) ranges from 0 to 450; the Y-axis (Weapon 2) ranges from 0 to 750. The plot looks very much like a plane, yet fits much better than the linear production function which is a plane.

The most interesting aspect of these varying production functions is the different interpretations one gets when cost is included in the analysis. To identify this relationship, the production function will be maximized subject to a cost constraint.

$$\text{Maximixe} \quad Y = A(\alpha W_1^{-P} + (1-\alpha)W_2^{-P})^{-V/P}$$

$$\text{Subject to } TC = C_1W_1 + C_2W_2$$

Note: TC is total funding, C_1 is the price of weapon one and C_2 is the price of weapon two.

OR

$$\text{Max } L = A(\alpha W_1^{-P} + (1-\alpha)W_2^{-P})^{-V/P} - \mu(C_1W_1 + C_2W_2 - TC) \quad (4.2)$$

Equation 4.2 is the Lagrangian. Next, the partials are taken and set equal to zero.

$$\frac{\partial L}{\partial W_1} = (-V/P)A(\alpha W_1^{-P} + (1-\alpha)W_2^{-P})^{(-V/P)-1}(-P)\alpha W_1^{-P-1} - \mu C_1 = 0 \quad (4.3a)$$

$$\frac{\partial L}{\partial W_2} = (-V/P)A(\alpha W_1^{-P} + (1-\alpha)W_2^{-P})^{(-V/P)-1}(-P)(1-\alpha)W_2^{-P-1} - \mu C_2 = 0 \quad (4.3b)$$

$$\frac{\partial L}{\partial \mu} = C_1W_1 + C_2W_2 - TC = 0 \qquad (4.3c)$$

Solving for μ in the first two equations and then setting them equal to each other yields the following relationship:

$$W_1 = [\alpha C_2/(1-\alpha)C_1]^{1/(P+1)}W_2 \qquad (4.4)$$

Substituting equation 4.4 into equation 4.3c gives the amount of W_2 required to maximize damage, the demand equation (equation 4.5).

$$W_2 = \frac{TC}{C_1[\alpha C_2/(1-\alpha)C_1]^{1/(P+1)} + C_2} \qquad (4.5)$$

The substitution coefficient is $\sigma = 1/(P+1)$. Since $P = -1$, there is absolute ease of substitution (σ = infinity). Within the demand equation 4.5 is a very important ratio (equation 4.6). Since σ is infinite, there are three potential results. If the value within the brackets of equation 4.6 is equal to one, then equation 4.5 simplifies to the form $W_2 = TC/[C_1 + C_2]$, and one buys an equal number of weapons one and two. If the ratio within the brackets is less than one, then equation 4.5 simplifies to $W_2 = TC/C_2$ and only weapon two is bought. If the ratio within the brackets is greater than one, the denominator goes to infinity causing no weapon two's to be bought, only weapon one.

$$[\alpha C_2/(1-\alpha)C_1]^{\sigma} \qquad (4.6)$$

The only case where the ratio can be equal to one is if the following holds:

$$\alpha/(1-\alpha) = C_1/C_2 \qquad (4.7)$$

Substituting the actual values implies that in order to buy a mix of weapons the ratio of the cost of weapon one to the cost of weapon two must be .52/.48 or 1.08. If the first partials of equation 4.1 are taken, which are the marginal products, and then divided by each other, the same result is gained. It is clear then, that the ratio of the intensities is the rate of technical substitution.

Since the ratio of the intensity parameters, $\alpha/1-\alpha$, is the slope of the isoquant, and the ratio of prices, C_1/C_2, is the slope of the budget curve, for the case where the ratio of the prices is equal to the ratio of the intensities, any combination of weapon one and weapon two is the best economic choice. The math suggested an equal amount of each weapon, but for this special case there are an infinite number of solutions. The decision to buy all of either weapon is one of these infinite number of solutions. Hence, for any ratio of prices, the decision to buy all of one weapon is the best economic choice.

The conclusion, that in all cases, one only buys one type of weapon and not a mix, does not seem to be like one's intuition of a strategic weapons problem. Unless both weapons possess similar capabilities across a range of different targets, one would not expect this answer. The weapons are generic, and no attempt was made to identify what a weapon most closely resembles since the data are only unclassified estimates. This conclusion hinges on the fact that the

weapons are perfect substitutes in economic terms. The application is that cost effective weapon decisions can be made based only on the price of the weapons and the intensity parameters of the CES production function.

2. THE POLYNOMIAL PRODUCTION FUNCTION. The polynomial is probably the most obscure of the production functions estimated in this study. The coefficients of the polynomial do not have the descript properties of the CES, and it takes some work to find its economic interpretation. The form of the polynomial for two variables is given in equation 4.8.

$$Y = \beta_0 + \beta_1 W_1 + \beta_2 W_2 + \beta_{12} W_{12} + \beta_{11} W_1^2 + \beta_{22} W_2^2 \qquad (4.8)$$

The β_0 term implies some ability to produce when all the inputs are zero. Obviously, this does not make economic sense. The function can be estimated without the β_0 term, but the fit is a little better with it. Since it is not very large, the β_0 term was left in the model. Equation 4.9 and 4.10 are the results of estimating the polynomial through multiple regression for the two variable case. Figure 4.2 is a three dimensional plot of equation 4.9.

Bias Design Estimated

$$Y = -21.8 + .91W_1 + .99W_2 - 2.4(10^{-4})W_1W_2 - 2.4(10^{-4})W_2^2 \qquad (4.9)$$

Variance Design Estimated

$$Y = -12.4 + .96W_1 + 1.06W_2 - 3.9(10^{-4})W_1W_2 - 1.4(10^{-4})W_1^2 - 3.8(10^{-4})W_2^2 \qquad (4.10)$$

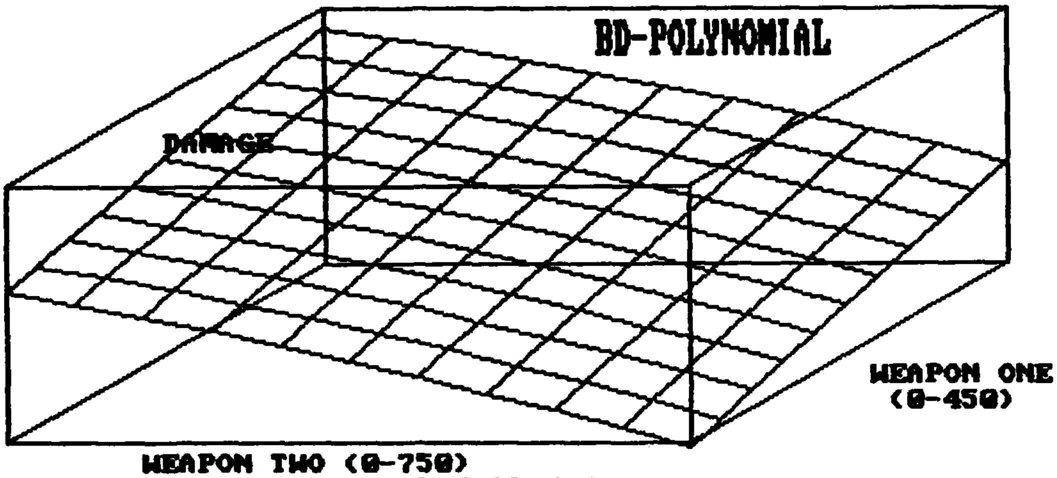

FIGURE 4.2

The polynomial is not homogeneous and therefore, the returns to scale cannot be measured. The elasticities of output can be determined though, using the standard definition:

$$E_{Y,X} = (X/Y)(\partial Y/\partial X)$$

It appears that because of scale and the non-homogeneous nature of the function, the output elasticities change for differing levels of production. Table 4.3 is the elasticities at the different extreme combinations of the inputs. The elasticities agree closely for both designs. The bias design, in general, has slightly larger values.

	INPUT LEVEL		BIAS DESIGN			VARIANCE DESIGN		
			ELAST	ELAST		ELAST	ELAST	
	W1	W2	W1	W2	SUM	W1	W2	SUM
	50	50	.62	.66	1.28	.54	.58	1.12
	50	700	.06	.76	.82	.06	.62	.68
	400	50	.93	.11	1.04	.84	.11	.95
	400	700	.35	.46	.91	.29	.33	.62
MEAN:	225	375	.37	.57	.94	.33	.51	.84

TABLE 4.3

If the polynomial was homogeneous, the elasticities would not be affected by scale changes. The Cobb-Douglas, for example, has elasticities of output that do not change for different levels of output. The CES elasticities of output do change at the varying levels of production, but always sum to the same value. As can be seen from table 4.3, the polynomial's elasticities do change and do not sum to the same value.

The ease of substitution is a difficult parameter to exact. Equation 4.11 is the formula used in estimating the elasticity of substitution (McFadden, 1963:75). One would expect it to be high given a knowledge of the production process (weapons producing damage), yet it is not infinite (implying perfect ease of substitution) as in the CES case.

$$\sigma_{ij} = (1/x_i f_i + 1/x_j f_j)/(-f_{ii}/f_i^2 + 2f_{ij}/f_i f_j - f_{jj}/f_j^2) \quad (i \neq j) \qquad (4.11)$$

The mean value for σ_{12} is one hundred twenty three; it ranged from fifteen to fifteen hundred. The low values occured at the maximum levels of the weapons.

Figure 4.3 is a plot of four isoquants. An isoquant is the combination of inputs that maintain a constant production. As can be seen from the figure, the isoquants are fairly flat, especially within the actual range of the weapons. An isoquant that is a straight line implies perfect ease of substitution. This figure supports the CES result of perfect ease of substitution.

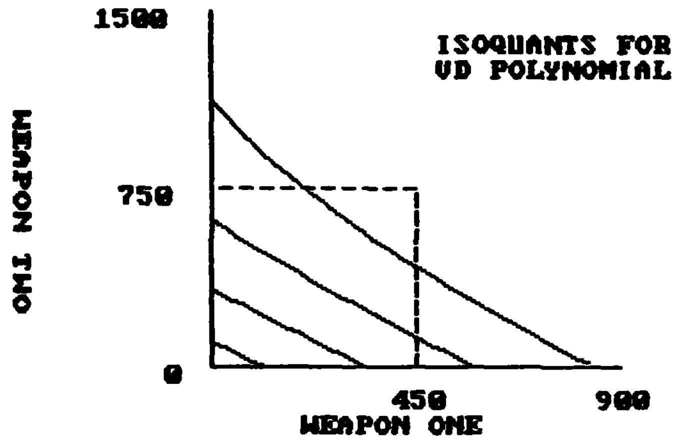

FIGURE 4.3

Maximizing the polynomial subject to cost gives the Lagrangian (equation 4.12).

$$\text{Max } L = \beta_0 + \beta_1 W_1 + \beta_2 W_2 + \beta_{12} W_1 W_2 + \beta_{11} W_1^2 + \beta_{22} W_2^2 - \mu(C_1 W_1 + C_2 W_2 - TC) \quad (4.12)$$

Taking the partial derivatives yields the following set of equations:

$$\frac{\partial L}{W_1} = \beta_1 + \beta_{12} W_2 + 2\beta_{11} W_1 - \mu C_1 = 0 \quad (4.13a)$$

$$\frac{\partial L}{W_2} = \beta_2 + \beta_{12} W_1 + 2\beta_{22} W_2 - \mu C_2 = 0 \quad (4.13b)$$

$$\frac{\partial L}{\partial \mu} = C_1 W_1 + C_2 W_2 - TC = 0 \quad (4.13c)$$

Eliminating μ in equations 4.13a and 4.13b gives the following relationship:

$$W_1 = \frac{\beta_2 C_1 - \beta_1 C_2 + W_2(2\beta_{22}C_1 - \beta_{12}C_2)}{2\beta_{11}C_2 - \beta_{12}C_1} \qquad (4.14)$$

Substituting into 4.13c gives the relationship that tells how many of either weapon one buys depending on the total funding, TC, and prices, C_i.

$$W_2 = \frac{TC(2\beta_{11}C_2 - \beta_{12}C_1) - \beta_2 C_1^2 + \beta_1 C_2 C_1}{2(\beta_{22}C_1^2 + \beta_{11}C_2^2 - \beta_{12}C_2 C_1)} \qquad (4.15)$$

It is not clear from looking at equation 4.15 what will happen when either total cost or an individual weapon cost changes. As an exercise, these equations, with the values from the polynomial, were tested on some fictional data. The use of fictional cost creates no problem since it is only measuring the relative effect of costs. Table 4.4 illustrates the case where the cost of the weapons are held the same, but there is an increase in funding.

FUNDING: 4000	C_1: 5.0	C_2: 4.0
BD-POLYNOMIAL	W_1: 244	W_2: 695
VD-POLYNOMIAL	W_1: 388	W_2: 515
FUNDING: 4200	C_1: 5.0	C_2: 4.0
BD-POLYNOMIAL	W_1: 362	W_2: 597
VD-POLYNOMIAL	W_1: 445	W_2: 494

TABLE 4.4

It is interesting to note that the number of weapons of type two decrease as funding is increased. In economic terms, this suggest an inferior good. An example of an inferior good might be potatoes in Ireland in the eighteenth century. Potatoes were needed for survival, they were a

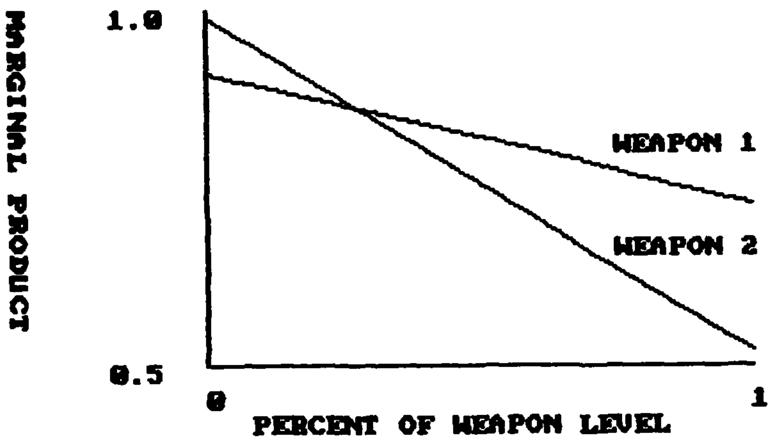

FIGURE 4.4

basic necessity. Yet, if income went up, people would buy less potatoes for food and more of other staples.

Is weapon two really an inferior good when compared with weapon one? A clear measure of the capability of a weapon is its marginal product. The marginal products are easily computed; they are simply the first partials of the function. Figure 4.4 is a graph of the first partials over the range of the weapons. Percent of weapon level refers to the percentage of weapons used within the limits of the weapon range. For example, 100 percent usage of weapon one is 450 weapons; 100 percent of weapon two is 750 weapons.

As the level of production increases, weapon one becomes a better weapon than weapon two. What is happening in the model is that weapon two represents a somewhat inaccurate but large megatonnage weapon. For soft targets, accuracy is not very important, and a larger payload (mega-

tonnage) achieves more destruction. As the level of production increases and more difficult, harder targets are attacked, weapon one, which represents good accuracy and small payload, performs better. Hard targets are more susceptible to accurate weapons than large payload weapons.

The actual feasible range of a nuclear weapons problem would include these harder targets. Therefore, weapon one is a superior weapon to weapon two in a realistic scenario.

Returning to economics, if weapon two is really an inferior good, the partial of W_2 with TC (equation 4.15a) should be negative.

$$\frac{\partial W_2}{\partial TC} = \frac{2\beta_{11}C_2 - \beta_{12}C_1}{2(\beta_{22}C_1^2 + \beta_{11}C_2^2 - \beta_{12}C_2C_1)} \qquad (4.15a)$$

Setting C_2 equal to one and substituting for the β values, one gets equation 4.15b. Setting C_2 equal to one does not change the problem in any way since it is only changing the units of measure.

$$\frac{\partial W_2}{\partial TC} = \frac{-2.8 + 3.9C_1}{-7.6C_1^2 + 7.8C_1 - 2.8} \qquad (4.15b)$$

Since weapon one is a better weapon than weapon two over most of the range, the only feasible solutions will be where the price of weapon two is less than the price of weapon one. Since the price of weapon one is greater than weapon two, and the price of weapon two is one, the numerator of equation 4.15b is positive. Now in order for equation 4.15b to be negative, the denominator must be negative.

It can be easily seen that for all values of C_1 greater than one (the price of C_2), the denominator is negative. Hence, the partial (equation 4.15b) is negative and weapon two is an inferior good.

How does this apply to the weapon problem? It has already been shown that weapon one is a better weapon compared to weapon two. Yet, if weapon one costs more than weapon two, not as many can be bought as weapon two. At lower funding levels, the most payoff can be accomplished using a larger number of cheaper weapons. As the funding level increases, more weapons will be exhausting the targets, and weapon one will give a better payoff. In other words, at lower funding levels quantity is more important than quality, and at higher levels quality is more important.

As for the effect of reducing the cost of a weapon, table 4.5 illustrates this with an example.

FUNDING: 4000	C_1: 5.0	C_2: 4.0
BD-POLYNOMIAL	W_1^1: 244	W_2^2: 695
VD-POLYNOMIAL	W_1^1: 388	W_2^2: 515
FUNDING: 4000	C_1: 5.0	C_2: 3.95
BD-POLYNOMIAL	W_1^1: 207	W_2^2: 750
VD-POLYNOMIAL	W_1^1: 380	W_2^2: 550
FUNDING: 4000	C_1: 4.9	C_2: 4.0
BD-POLYNOMIAL	W_1^1: 368	W_2^2: 549
VD-POLYNOMIAL	W_1^1: 427	W_2^2: 476

TABLE 4.5

RATE OF TECHNICAL SUBSTITUTION OF W_j FOR W_i AT MEAN (RTS_{ji})

	RTS_{21}	RTS_{31}	RTS_{32}
Polynomial-BD	1.310	2.471	1.887

INTENSITY OF I DIVIDED BY J (I_{ji})

	I_{21}	I_{31}	I_{32}
CES	1.309	2.492	1.904

TABLE 5.3

the production process. Since there is perfect ease of substitution, the inputs can be freely substituted for each other. If the inputs can be freely substituted, the technology of the production process will take the better producing input, hence, the intensities measure comparable marginal output (or product). If the inputs were all priced the same, one would only use the best output produc[illegible] input. This all hinges on the fact that a mix of inputs is not needed for production, i.e., the perfect ease of substitution assumption.

A very interesting implication is that the intensity parameters of the CES can now be used to make decisions regarding the mix of weapons to buy. The critical points are given in equations 5.3a-5.3c. Note that C_i refers to the price of weapon i, and the α, β, and $1-\alpha-\beta$ terms are the intensity parameters of the CES.

$$C_2 = \frac{\beta}{\alpha} C_1 \qquad (5.3a)$$

$$Y = 17(.46W_1 + .35W_2 + .19W_3)^{.65} \quad (5.2)$$

The most interesting observation is that the returns to scale is much less than one. This problem is characterized by noticeably diminishing returns to scale. This agrees with earlier observations in the chapter that there are fewer targets for the total weapons. Many of the weapons in this problem are attacking very difficult targets or are being doubled up on other targets. The effect is that there is considerably less return at higher levels of output. The CES assumes that the returns to scale is constant at 0.65 across the range of input combinations. This is not really true, the CES has in effect averaged the returns to scale across the range.

The distribution parameters, or intensity parameters, have the same interpretation as in the two variable case. They measure comparative mean marginal products. The term comparative is used because they are not the mean marginal products of one weapon, but of some unknown units of a weapon. Dividing the intensities by each other is the same as computing the marginal products and dividing them by each other. The result is the rate of technical substitution. Table 5.3 illustrates this property.

The fact that there is perfect ease of substitution is why the intensities exhibit mean marginal values. In more typical economic problems, the intensity of an input is a measure of how intensive that input is in the technology of

other near the mean, and not more than twenty-five percent different at the extreme points.

ECONOMIC VALUES AT THE MEAN

FUNCTION	MARGINAL PRODUCTS W_1	W_2	W_3	OUTPUT ELASTICITIES W_1	W_2	W_3
Polynomial-BD	.72	.55	.29	.21	.25	.20
Polynomial-VD	.70	.56	.33	.21	.26	.23
CES-BD	.70	.56	.28	.21	.25	.20

TABLE 5.2

The next two sections will more closely evaluate the economic interpretation of these functions.

1. <u>THE CES PRODUCTION FUNCTION</u>. Since the CES is the easiest to interpret in economic terms, it is evaluated first. The form of the CES for three variables is given in equation 5.1

$$Y = A(\alpha W_1^{-P} + \beta W_2^{-P} + (1-\alpha-\beta)W_3^{-P})^{-V/P} \qquad (5.1)$$

The interpretation of the CES parameters are the same as in the two variable case. Using a nonlinear estimation technique to evaluate the parameters resulted in the P parameter taking on the value, P = -1.05, which is outside the range for P. A P value of negative one implies perfect ease of substitution. Therefore, P was set to its lower limit, negative one, and the nonlinear estimation technique was applied on the remaining parameters. The estimated equation is given below (equation 5.2). The function was estimated using the bias design points.

is obvious since there are three variables instead of two. But also, the number of targets (not target types) was increased for the three variable case. Because the increase in weapons was proportionately greater than the increase in targets, the result is that there are less targets per total weapons involved than in the two variable problem. The effect in the model is to increase some right hand side (RHS) values. Therefore, the two problems are entirely different, although both use the same methodology to maximize damage expectancy. This poses no problems in the analysis, since the objective is not to analyze some specific weapons, but rather to study this methodology of applying RSM and economics, and the information gained through using this methodology to determine cost effective weapon choices.

B. INTERPRETATION VALUE

A simple comparison at the mean shows that the functions give similar economic values (Table 5.2). A more convincing argument is to look at the economic values over a wide range of the inputs. This was also accomplished during the research. The results over the range of input combinations for both functions agreed quite closely on these basic economic measures. 'Quite closely' is a subjective measure of the agreement among the functions. A more objective judgement is that the values are within five percent of each

V. ANALYSIS OF THE THREE VARIABLE RESULTS

A. PREDICTION VALUE

Similar to the two variable analysis, for the three variable case, there are two important elements in choosing a function to analyze the surface, prediction ability and interpretational ability. In terms of prediction value, the polynomial is clearly the better fitting function (table 5.1). Regardless of prediction measure of merit, the polynomial provides a better fit to the surface. The next best fitting function is the CES. Surprisingly, all other functions had percent fits around 90 percent. Based on these results and those of chapter IV the polynomial and the CES are the only two functions evaluated in this chapter.

FUNCTION	RANDOM DESIGN DATA POINTS SDS	PERCENT FIT TO SURFACE
Polynomial (VD)	5446	0.98
Polynomial (BD)	6013	0.98
CES (BD)	45706	0.93
Linear (BD)	86875	0.90
Linear (VD)	87818	0.90
MNH (BD)	128150	0.91
Cobb-Douglas (BD)	228665	0.88

Table 5.1

Before preceding into the interpretation of the functions, the question arises; why doesn't the other production functions fit as well as in the two variable case? The primary reason is that the LP problems are different. This

closely on some major economic interpretations. First, the production process has high ease of substitution. Secondly, there are diminishing returns to scale. Thirdly, the analysis leads one to buy all of one type of weapon for most cost relationships. Fourthly, weapon one is a generally superior weapon compared to weapon two.

Of the production functions, if prediction and interpretation are of similar importance, the CES would seem to be the best to use. It fits very well, 98% to the random surface, and has very descriptive properties.

The C-D does fit fairly well away from the axis. The elasticities are good measures of the mean percent change in output for one percent changes in the input. Still, it would appear that one of the preceding functions could give a better fit and provide as much economic insight into the problem, if not more.

6. THE MULTIPLICATIVE NON-HOMOGENEOUS PRODUCTION FUNCTION. The MNH is similar to the Cobb-Douglas, but also allows for variable elasticities of substitution. The form of the MNH is given in equation 4.24.

$$Y = AW_1^{(\alpha + \theta LN(W_1))} W_2^{\beta} \tag{4.24}$$

The MNH, when estimated using the variance design, was an extremely poor fit to the surface of the random data points; on average the error was 45 percent. The bias design was somewhat better; the fit was 88 percent, which translates into a 12 percent error on average. The estimated parameters of the MNH for the bias design are given in equation 4.25.

$$Y = 0.001W_1^{(1.83-0.26LN(W_2))} W_2^{1.90} \tag{4.25}$$

Since the function is not a very good fit to the surface, it will not be evaluated any further.

C. SUMMARY

In conclusion, a number of observations can be made. Among the functions that fit the surface well, the polynomial, the CES, the linear, and the ANH, all correlate

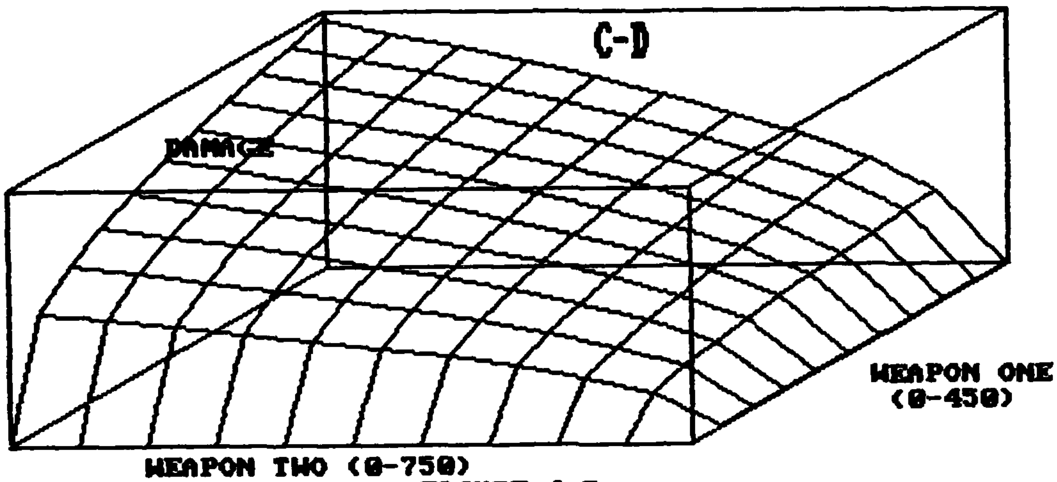

FIGURE 4.5

elasticities are constant, independent of the production level. The output elasticities added together measure the returns to scale (.87). The returns to scale in the C-D case is a little less than the CES measured (.94). They do both agree that there is diminishing returns to scale.

Figure 4.6 is a three dimensional plot of equation 4.23. The most noticeable inconsistency of the C-D is seen in figure 4.6, that is, the C-D requires positive levels of both inputs before there is any output. Not only does this disagree with the problem but also with the implications of the preceding production functions. The main implication of the other production functions has been that there is perfect ease of substitution or at least very high. The C-D does not agree with this. It implies that when the costs are varied, one always buys a mix of weapons. This can be seen from figure 4.6; one never buys zero of a weapon.

It is not necessary to set up the Lagrangian in the simple case of a linear model. Since the substitution is infinite, it is merely a matter of finding the slope of the isoquants to compute the ratio of costs needed to buy a mixture or buy all of one weapon. If the ratio of the prices C_1/C_2 exceeds MP_1/MP_2 (where MP represents the marginal products), then one buys all weapon two. If the ratio of C_1/C_2 is less than MP_1/MP_2, one buys all weapon one. If the ratios of costs and marginal products are equal, then any combination of weapons at the costs are appropriate to maximize damage expectancy.

5. THE COBB-DOUGLAS PRODUCTION FUNCTION. The C-D also has very descriptive properties and a simple form. It has been used extensively in economics. The form of the C-D is given in equation 4.22.

$$Y = AW_1^{\alpha}W_2^{\beta} \tag{4.22}$$

The C-D was estimated using the bias design. The C-D is linear in the logs. Therefore, since the bias design is orthogonal for functions of order one, the parameters are not correlated. Equation 4.23 is the estimated function.

$$Y = 3.6W_1^{0.33}W_2^{0.54} \tag{4.23}$$

The values, 0.33 and 0.54, are the output elasticities for weapon one and two respectively. These measure the percent change in output for a one percent change in the input. In contrast to the other functions, the output

$$Y = \beta_0 + \beta_1 W_1 + \beta_2 W_2 \qquad (4.19)$$

The linear function was estimated using the bias design, which is orthogonal for models of order one, and is given as:

$$Y = 25.2 + 0.82W_1 + 0.75W_2 \qquad (4.20)$$

The β_1 and β_2 terms (0.82 and 0.75) represent the marginal products of weapons one and two respectively. The marginal products have a very obvious definition from the equation: the increase in output for a one unit increase in the input. Additionally, the marginal products are constant. The other production functions have decreasing marginal products, a more intuitive result. If the β_0 term had been zero, then the linear function would be homogeneous of degree one. Since β_0 is equal to 25.2, the model is quasi-homogeneous of degree one for high levels of the inputs, but clearly nonhomogeneous for lower levels of the inputs. Additionally, the isoquants for equation 4.20 are cleary linear and therefore, the substitution is infinite. This is the same result as in the CES model. The polynomial supports high ease of substitution, but not perfect ease as the linear and CES are interpreted.

The elasticities are easily computed by equation 4.21 as the marginal product of weapon 'i' times the level of weapon 'i' divided by the output at that level of the inputs.

$$E_{Y,W_i} = MP_i W_i / Y \qquad (4.21)$$

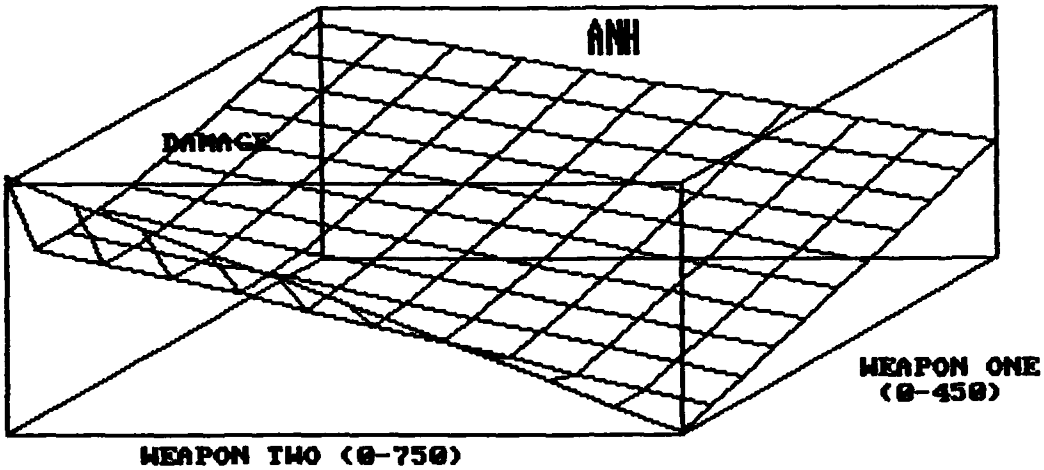

FIGURE 4.5

that both have a positive interaction term. There is multicollinearity in both design estimates.

BIAS DESIGN ESTIMATED

$$Y = 7.8 + 0.53W_1 + 1.15W_2 + 0.079W_1LN(W_2) - 0.085W_2LN(W_1) \tag{4.17}$$

VARIANCE DESIGN ESTIMATED

$$Y = 27.9 + 0.78W_1 + 0.74W_2 + 0.023W_1LN(W_2) - 0.026W_2LN(W_1) \tag{4.18}$$

4. THE LINEAR PRODUCTION FUNCTION. The linear model is probably the simplest of all the models to understand. While the coefficients of a linear model can be obscure due to differences in the magnitude of the inputs, this is not the case in this study. A one unit increase in either input represent a one weapon increase. The form of the linear is given below (equation 4.19).

$$Y = \beta_0 + \beta_1 W_1 + \beta_2 W_2 + \beta_{12} W_1 LN(W_2) + \beta_{21} W_2 LN(W_1) \tag{4.16}$$

The ANH is basically a linear model with two interaction terms. This particular model was probably the most difficult to estimate, especially for more than two variables. The two interaction terms produced opposite signs when estimated using either the bias design or the variance design. In all other cases of interactions, the effect has been negative. The interpretation of this model with a nonnegative interaction term is suspect. The results do not agree with intuition in certain places. The major difficulty is for the case of a large number of weapon two and a small number of weapon one; the marginal product of weapon one is negative. There should be no negative marginal products at this level of the inputs, in fact, the other production functions estimate the marginal product of weapon one at anywhere from 0.66 to 0.82. While the ANH strongly agrees with the other functions in the other ranges for economic values, this is one area where it is extremely suspect.

Because of these problems with the ANH, it will not be discussed in terms of economic meaning. For completeness, the actual estimated function is given in equation 4.17 and a graph of the function in figure 4.5. The variance design estimated equation is given in 4.18 for comparison purposes. Note that the linear coefficients are quite different, and

does not seem to agree with strategic nuclear military judgement. The reason is that the LP model has as its objective, damage expectancy. It does not model deterrence. Risk would be a main factor in the results if it did, and would probably lead to conclusions where a combination of weapons is the best choice. What does this say about simple models of this type used in a strategic decision making evironment? They give economic reasons for choosing one weapon over another, but a more complicated model may be needed to give sound advice in force composition. This is not to say that the model could not be used to examine force increases. Given that the strategic force is already a mix of weapons, what is the best economic choice for increasing the capability of the force? Here the results imply that this best choice will be to buy all of one weapon. The results could also be used for examining the particular weapons in one of the legs of the triad. For example, the B-1 bomber and Stealth bomber could be analyzed for best choice as an addition to the present bomber leg of the triad.

3. <u>THE ADDITIVE NON-HOMOGENEOUS PRODUCTION FUNCTION</u> (ANH). The ANH was originally developed for a very specific telecommunications problem where there was some aprior information that the problem did not have constant elasticities of substitution. The returns to scale changed over time and the elasticities of output also changed. The form of the ANH is given in equation 4.16.

As one would expect, reducing the cost of either weapon causes an increase in the number of that weapon purchased. Further, a reduction in the cost of any weapon also causes less of the other weapon to be bought. A more rigorous proof is to take the partial of W_2 with respect to either C_1 or C_2. The resulting equation is a very difficult equation to understand (equation 4.15c).

$$\frac{\partial W_2}{\partial C_2} = \frac{(\beta_{22}C_1^2+\beta_{11}C_2^2-\beta_{12}C_1C_2)[TC(2\beta_{11}+\beta_1C_2)] - [TC(2\beta_{11}C_2-\beta_{12}C_1)-\beta_2C_1^2+\beta_1C_2C_1]}{2(\beta_{22}C_1^{\;2} + \beta_{11}C_2^{\;2} - \beta_{12}C_2C_1)^2} \tag{4.15c}$$

Since the denominator is squared and hence, always positive, one need only show that the numerator is negative to prove the assertion that a reduction in price will cause an increase in purchase. Unfortunately, multiplying the numerator out yielded a very complicated expression. Even setting a weapon price to one and working in the feasible range did not simplify the expression enough to make any conclusions about the sign.

One last comment in regard to the polynomial, unlike the CES, there is a range of relative cost ratios where one would buy a combination of the two weapons. Yet, as was implied in the plots of the isoquants (figure 4.3), this range is small, lending credence to the CES result that it is economically efficient to buy all of one weapon to get the most destruction for a given funding level. This result

$$C_3 = \frac{1-\alpha-\beta}{\alpha} C_1 \qquad (5.3b)$$

$$C_3 = \frac{1-\alpha-\beta}{\beta} C_2 \qquad (5.3c)$$

If the above equalities hold, any combination of weapons would be the best economic buy. This is true because the ratio of the prices for any two inputs are equal to the slope of the isoquants (the ratio of the intensities) for those two inputs. The results are similar to those attained in the two variable case. The best economic decision is to buy any combination of weapons, including the choice to buy all of one weapon. The case where the above equalities hold will be rare, more likely will be buy decisions where none of the above equalities hold. These cases as well as the above case are all summarized in figure 5.1.

BUY ALL W_1	BUY ALL W_2	BUY ALL W_3
$C_1 \leq \frac{\alpha C_2}{\beta}$	$C_2 \leq \frac{\beta C_1}{\alpha}$	$C_3 \leq \frac{(1-\alpha-\beta) C_1}{\alpha}$
$C_1 \leq \frac{\alpha C_3}{1-\alpha-\beta}$	$C_2 \leq \frac{\beta C_3}{1-\alpha-\beta}$	$C_3 \leq \frac{(1-\alpha-\beta) C_2}{\beta}$

FIGURE 5.1

If the two conditions under a particular buy decision hold, then that decision is as good as any other possible buy decision. Other buy decisions could also hold and they would represent an equally best economic choice. Rarely

though, will more that one of the above decisions be true. Most ratios of prices will not be equal to the ratio of the intensities. In all these cases, only one of the above decisions will have both conditions holding, and that decision will be the best economic choice.

The CES does have some nice interpretation values in this problem. The fit is not as good as the polynomial (93% versus 98%), but it allows for much easier interpretation. Of course, one must decide if the CES percent fit of 93% captures the surface well enough to be able to make decisions using it.

2. <u>THE POLYNOMIAL PRODUCTION FUNCTION</u>. The polynomial function increases quite dramatically in terms with the addition of the third variable. The larger (more variables) a problem contains, the more difficult the polynomial is to evaluate. The estimated function, using the variance design, is given in equation 5.6. The variance design polynomial was used instead of the bias design because both fit equally well, and the variance design has no multicollinearity.

$$\begin{aligned} Y = -40 &+ 1.2W_1 + 1.1W_2 + 0.9W_3 \\ &-4.5(10^{-4})W_1W_2 - 2.6(10^{-4})W_1W_3 - 3.9(10^{-4})W_2W_3 \\ &-3.8(10^{-4})W_1^2 - 3.2(10^{-4})W_2^2 - 3.3(10^{-4})W_3^2 \end{aligned} \quad (5.6)$$

The coefficients of the linear terms are positive and give the marginal products of the weapons at low input

levels. They also give the maximum possible output for a weapon. As the level of the inputs increase, the interaction terms and quadratic terms begin to decrease the return on a weapon. In fact at a level of weapons beyond the feasible range of weapons used in this problem, the output would actually begin to decrease.

The coefficients of the interaction and quadratic terms are also slightly affected by the scale of the problem. The feasible range of weapon one is 0-450; weapon two is 0-750; weapon three is 0-1040. As can be seen from the ranges, at maximum level of the inputs, weapon three has over twice as many weapons as weapon one. Therefore, even though the coefficients of the higher order terms involving weapon three are smaller than the coefficient of weapon one, at the higher input levels weapon three has a larger negative effect on its marginal output than weapon one. The same argument can be used in comparing weapon two to weapon one. The bottom line is that due to the scale effects involved in the coefficients of the polynomial function, it is difficult to interpret the function without a lot more work and a lot of examples.

In order to evaluate the results of adding cost to the polynomial function, a lagrangian needs to be set up. Due to the complexity of the problem, a more simplified approach will be used to study the effect of costs. By holding one of the variables at some constant level, the problem trans-

fers into a two variable problem. The results of Chapter IV can then be used.

Since the CES confirmed that the problem exhibits perfect ease of substitution, and intution of a weapons problem suggests perfect ease of substitution, this will be assumed. This assumption implies that the isoquants are flat (straight lines), meaning that in all cases, only one weapon will be bought. The actual estimated values for the elasticities of substitution for the polynomial were high; so this agrees with the assumption.

Once perfect ease (or at least high ease) of substitution is assumed, most buy decisions will buy only one weapon. Therefore, in analyzing two weapons against each other, the other weapon can assumed to be zero.

The most interesting conclusion from the two variable case is that one of the weapons was an inferior good. This conclusion is tested in the context of the three weapon problem to see if it also holds here.

The most likely candidate for an inferior good is weapon three. It is definitely a poorer weapon in capability than weapons one and two. Setting weapon two to zero and evaluating the partial developed in the previous chapter (equation 4.15a) gives equation 5.7. Note that C_i is the price of weapon i, and TC is the total funding.

$$\frac{\partial W_3}{\partial TC} = \frac{-7.6C_3 + 2.6C_1}{2(-3.8C_3^2 - 3.3C_1^2 + 2.6C_1C_3)} \tag{5.7}$$

Restricting $C_3 < C_1$, keeps the problem within the feasible range. Since weapon one is superior to weapon three, none of weapon three would be bought if it cost more than weapon one. In order for equation 5.7 to be negative and prove the inferior good hypothesis, either the numerator is positive and the denominator is negative or vice a versa. Changing the units of cost such that $C_1 = 1$, the following relationship must hold for the inferior good hypothesis:

$$\boxed{\begin{array}{c} -7.6C_3 + 2.6 > 0 \\ \\ \text{AND} \\ \\ -3.8C_3^2 + 2.6C_3 - 3.3 < 0 \end{array}} \quad \text{OR} \quad \boxed{\begin{array}{c} -7.6C_3 + 2.6 < 0 \\ \\ \text{AND} \\ \\ -3.8C_3^2 + 2.6C_3 - 3.3 > 0 \end{array}}$$

The first set of inequalities holds for $C_3 < 2.6/7.6$ or 0.34. The second set of inequalities never holds. The conclusion is that weapon three is an inferior good at prices less than 34 percent of weapon one. One question stands out, why does weapon three lose its inferior good quality as its price increases? It appears that at levels where the price is higher than 34 percent of weapon one, the problem is infeasible, that is, negative amounts of weapons are being bought. To test this possibility is beyond the scope of this thesis. One interesting point to be included here is that the CES would not buy weapon three at prices greater than 40 percent of weapon one (Intensity of W_3 divided by the Intensity of W_1). There may be some correlation between these two results. The difference may be that

the polynomial does not have perfect ease of substitution, but only high ease of substitution.

Performing exactly the same above analysis for weapon two and weapon one, one finds that weapon two is an inferior good for prices of weapon two 59 percent or less than weapon one. For prices greater than this ratio, weapon two is not an inferior good. Again, it could be that prices greater than this ratio are infeasible points in the problem; this was not determined. The corresponding ratio of prices break point for the CES was 76 percent (Intensity of W_2 divided by the Intensity of W_1).

C. SUMMARY

The similarities between the polynomial and the CES are that they both advocate buying a best weapon as opposed to a mix, and that they both support the fact that the weapons are very interchangeable, substitutable. Perhaps this is a fault of the model. The model may be so simple that it has not captured the essential aspects of a strategic weapons problem, in which case, the analysis has revealed a potential limitation of using this type of model.

There are a lot of similarities in the useful information gained from the two and three variable cases. The next chapter will attempt to succinctly summarize the value of this research.

VI. CONCLUSIONS AND RECOMMENDATIONS

A. OVERVIEW

This research represents a methodology for identifying cost effective mixes of not only strategic forces, but of potentially any type of weapon problem. In fact, the results apply to any problem using a deterministic model to generate data. The methodology is simply the application of economics and Response Surface Methodology (RSM) to deterministic models.

In this thesis, the deterministic model reflected a nuclear exchange problem. Using RSM to fit economic production functions to the surface of the nuclear exchange model and Lagrangian techniques to examine the effect of cost on these economic functions, allowed the identification of cost effective buys of weapons. In the nuclear exchange model used in this study, the cost effective choices were easily determined by simple buy relationships that developed out of one of the economic production functions (the CES). These simple buy rules allow for the determination of what weapon mix to buy based on only the relative prices of the weapons and the parameters of the CES production function.

B. SPECIFIC RESULTS

Five production functions, common in economics, were used in this research. They are the Constant Elasticity of Substitution function (Arrow and others, 1961), the Cobb-

Douglas, a Multiplicative Non-Homogeneous (Vinod, 1972), an Additive Non-Homogeneous (Sudit, 1973), and a Linear function. These five functions and a polynomial of order two were successfully fitted to the response surface of a production process using RSM. The production process, as mentioned above, is an unclassified nuclear exchange model. The input variables represent a particular type of warhead; the output or production is damage expectancy.

Of the six functions fitted to the response surface, three fit very well. These functions are the polynomial, the CES, and the Linear. Interpreting these functions gave insight into the production process that was not apparent at the beginning of the research. Additionally, the coefficients of the economic production functions provide information that is obscured by the coefficients of the polynomial. The ease of interpretation of the economic functions, such as the CES, provide for a better vehicle for analysis and decision-making than the polynomial.

The polynomial production function provided a very good fit in both of the cases studied in this research, a two variable case and a three variable case. The good fit of the polynomial may be explained by the fact that the polynomial is a portion of the Taylor expansion of any function. While the polynomial has good prediction value, the interpretation takes more work to understand than the economic functions used in the study. The coefficients of the poly-

nomial do not have any direct relationship to the coefficients of the economic production functions. The best that can be said is that the coefficients of the linear terms in the polynomial represent a maximum marginal product of the inputs. Since all the interaction terms and quadratic terms are negative, these serve to reduce the marginal product as the input levels increase. It is very difficult to tell exactly what degree of negative effect is caused by the terms. The best alternative is to compute the first partials to interpret the polynomial. The first partials represent the marginal products and can be easily computed for the polynomial.

The CES production function also fits very well in the context of this problem, although not as well as the polynomial in the three variable case. The CES is an easy function for understanding the coeffients. The returns to scale is evident in the function and agrees with intuition that the problem is one of diminishing returns, although the CES only gives the average of the effect of returns to scale over the range. Also, the CES points out clearly the high ease of substitution in the problem which is not entirely evident from the polynomial function. This fact is probably the major cause why the analysis of the results shows that in all cases the best choice, in economic terms, is to buy only one weapon. Additionally, the coefficients of the CES gave good comparative results of the average marginal value

of a weapon. Finally, these comparative values could be used to make quick and simple cost decisions regarding weapon buys.

The research showed that economic production functions can be easily fitted to the response surface of a deterministic model and that the information gained from the fitting of these economic production functions can provide insight into the production process that will be useful to a decision maker. The information concerning returns to scale, ease of substitution, and the effect of cost on the problem is easily extracted from the economic production functions. The CES illustrated these results the best.

Also, the research was helpful because the use of different types of production functions gave insight into the problem which may not have been discovered had only one type of function been used. For example, the use of other production functions highlighted possible model inconsistencies. The result that the best choice is to buy only one weapon is not an obvious fact for a strategic weapons problem where emphasis has usually been placed on a mix of weapons (e.g. the Triad). Possibly, the model is too simple and needs to take into account the value of deterrence and risk, possibly through adding other constraints or goal programming. Additionally, the model need not only be applied to problems of force composition, but also where to spend additional funds in the pursuit of a more effective

force. In other words, what weapon is the best choice for increasing the effectiveness of the strategic force? The results of this research would allow for the determination of the most cost effective strategic weapon to be added to the existing arsenal.

C. RECOMMENDATIONS

This work provides a foundation for the use of economic models in the analysis of deterministic models of production processes. The simple buy decision rules developed from the CES production function can be a useful tool for the decision maker in cases where the inputs are substitutable. An interesting question would be to examine the results when applied to a more complex weapon problem. Instead of starting out with only a few variables, all the variables could be used, and an actual in depth analysis of the results using some of these economic production functions could be accomplished. The methodology as proposed in this study would choose an experimental design based on the Box and Bhenken Designs, run the designs for the deterministic model, fit the economic production functions used in this study, and apply a Lagrangian or nonlinear optimization routine to analyze the effect of a cost constraint.

Further work in this area would include the development of bias minimizing experimental designs which are also orthogonal. These would be a tremendous help for analyzing the results of RSM when applied to deterministic models.

This research has provided a methodology for the use of economics, RSM, and deterministic modelling. The use of this methodology has great potential for understanding problems more effectively and for giving the decision maker useful information regarding cost effective decisions.

Appendix A

This appendix lists the programs required to generate and run the Linear Programming model. A detailed description of the model can be found in Graney, 1984, appendix A.

The first program is the control cards required by the linear programming package, "Multi-Purpose Optimization System", MPOS, to execute the problem. The second program is a fortran code program to generate the data matrix for the LP program.

```
**** First Program ****

*THIS IS THE CONTROL CARD DECK FOR MPOS AEM
TITLE MCN
ARSENAL EXHANGE PROBLEM
MINIT
VARIABLES
X1 TO X100
PACKED
MAXIMIZE
CONSTRAINTS 15
+++++++++++++++
FORMAT
(2X,I3,2X,I3,2X,F10.5)
REWIND
READ MN
OPTIMIZE


**** Second Program *****

      PROGRAM CREATE2
      DIMENSION AIJ(0:15,0:100),CFIJ(0:15,0:100),CVIJ(0:15,0:100)
C
C
C     THIS PROGRAM CREATES THE DATA MATRIX FOR AN AGGREGATE ARSENAL
C     EXCHANGE MODEL TO BE SOLVED BY 'MPOS'.  THE MODEL HAS TEN
C     TARGET CATEGORIES AND FIVE WEAPON CLASSES.  THE MODEL ALLOCATES
C     A SINGLE TYPE WEAPON AGAINST A PARTICULAR TARGET.  THE CURRENT
```

```
C      OBJECTIVE(ROW 0) MAXIMIZES THE DAMAGE EXPECTED(DE) TO THE
C      TOTAL TARGET BASE.  THE MODEL ALLOWS A SINGLE WEAPON OR A
C      PAIR OF SAID WEAPONS TO BE ALLOCATED TO A SINGLE TARGET.
C
C      NOTE: THE RIGHT-HAND SIDE VALUES(RHS) ARE IN COL. ZERO AND
C            THE OBJECTIVE COEFF. ARE IN ROW ZERO
C
C
C
C
C
C      INITIALIZE MATRIX AIJ TO ALL ZEROS.
C
       DO 200, J=0,100
        DO 100, I=0,15
         AIJ(I,J)= 0.0
 100    CONTINUE
 200   CONTINUE
C
C
C      BUILDING TARGET CONSTRAINT SET(ROWS 1 TO 10 )
C
       DO 400, II=0,80,20
        DO 300, I=1,10
         AIJ(I,(2*I+II))= 1.0
         AIJ(I,(2*I-1+II))= 1.0
 300    CONTINUE
 400   CONTINUE
C
C
C      BUILD WEAPON CONSTRAINT SET(ROWS 11 TO 15)
C
C
       JA=1
       JB=19
C
       DO 800, I=11,15
         DO 600, J=JA,JB,2
          AIJ(I,J)= 1.0
          AIJ(I,(J+1))=2.0
 600     CONTINUE
C
        JA=JA+20
        JB=JB+20
C
 800   CONTINUE
C
C
C      SET RHS (COL.0  ROWS 0 THRU 10) FOR OBJECTIVE AND NUMBER OF
C          TARGETS IN EACH CATEGORY.
C
```

AD-A151 773 A METHODOLOGY FOR IDENTIFYING COST EFFECTIVE STRATEGIC FORCE MIXES(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGINEERING T W MANACAPILLI DEC 84 2/2

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END

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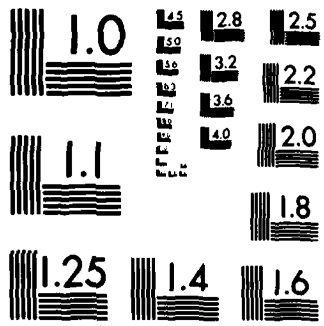

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

```
      AIJ(0,0)=    0.0
      AIJ(1,0)=  140.0
      AIJ(2,0)=  215.0
      AIJ(3,0)=  450.0
      AIJ(4,0)= 1000.0
      AIJ(5,0)=  200.0
      AIJ(6,0)=   20.0
      AIJ(7,0)=  150.0
      AIJ(8,0)=  100.0
      AIJ(9,0)=  430.0
      AIJ(10,0)= 520.0
C
C
C     SET PK'S IN OBJECTIVE FUNCTION(ROW 0, COLS. 1 TO 100)
C
      AIJ(0,1)=    0.84797        AIJ(0,38)=    0.92038
      AIJ(0,2)=    0.97689        AIJ(0,39)=    0.81872
      AIJ(0,3)=    0.85000        AIJ(0,40)=    0.96714
      AIJ(0,4)=    0.97750        AIJ(0,41)=    0.44306
      AIJ(0,5)=    0.83249        AIJ(0,42)=    0.68982
      AIJ(0,6)=    0.97194        AIJ(0,43)=    0.84164
      AIJ(0,7)=    0.47032        AIJ(0,44)=    0.97492
      AIJ(0,8)=    0.71944        AIJ(0,45)=    0.14544
      AIJ(0,9)=    0.67217        AIJ(0,46)=    0.26973
      AIJ(0,10)=   0.89253        AIJ(0,47)=    0.02663
      AIJ(0,11)=   0.84999        AIJ(0,48)=    0.05255
      AIJ(0,12)=   0.97750        AIJ(0,49)=    0.08063
      AIJ(0,13)=   0.85000        AIJ(0,50)=    0.15475
      AIJ(0,14)=   0.97750        AIJ(0,51)=    0.51804
      AIJ(0,15)=   0.85000        AIJ(0,52)=    0.76771
      AIJ(0,16)=   0.97750        AIJ(0,53)=    0.84982
      AIJ(0,17)=   0.84660        AIJ(0,54)=    0.97744
      AIJ(0,18)=   0.97647        AIJ(0,55)=    0.84097
      AIJ(0,19)=   0.84979        AIJ(0,56)=    0.97471
      AIJ(0,20)=   0.97744        AIJ(0,57)=    0.22121
      AIJ(0,21)=   0.76716        AIJ(0,58)=    0.39349
      AIJ(0,22)=   0.94579        AIJ(0,59)=    0.52353
      AIJ(0,23)=   0.85000        AIJ(0,60)=    0.77298
      AIJ(0,24)=   0.97750        AIJ(0,61)=    0.36172
      AIJ(0,25)=    0.61073       AIJ(0,62)=    0.59260
      AIJ(0,26)=    0.84847       AIJ(0,63)=    0.82551
      AIJ(0,27)=    0.15429       AIJ(0,64)=    0.96955
      AIJ(0,28)=    0.28478       AIJ(0,65)=    0.11517
      AIJ(0,29)=    0.37563       AIJ(0,66)=    0.21708
      AIJ(0,30)=    0.61016       AIJ(0,67)=    0.02402
      AIJ(0,31)=    0.83104       AIJ(0,68)=    0.04747
      AIJ(0,32)=    0.97145       AIJ(0,69)=    0.04995
      AIJ(0,33)=    0.85000       AIJ(0,70)=    0.09741
      AIJ(0,34)=    0.97750       AIJ(0,71)=    0.40469
      AIJ(0,35)=    0.85000       AIJ(0,72)=    0.64561
      AIJ(0,36)=    0.97750       AIJ(0,73)=    0.84494
      AIJ(0,37)=    0.71782       AIJ(0,74)=    0.97596
```

```
      AIJ(0,75)=    0.83678       AIJ(0,88)=    0.19496
      AIJ(0,76)=    0.97336       AIJ(0,89)=    0.26052
      AIJ(0,77)=    0.17125       AIJ(0,90)=    0.45317
      AIJ(0,78)=    0.31317       AIJ(0,91)=    0.80019
      AIJ(0,79)=    0.40083       AIJ(0,92)=    0.96008
      AIJ(0,80)=    0.64099       AIJ(0,93)=    0.85000
      AIJ(0,81)=    0.72773       AIJ(0,94)=    0.97750
      AIJ(0,82)=    0.92587       AIJ(0,95)=    0.85000
      AIJ(0,83)=    0.85000       AIJ(0,96)=    0.97750
      AIJ(0,84)=    0.97750       AIJ(0,97)=    0.60865
      AIJ(0,85)=    0.48420       AIJ(0,98)=    0.84685
      AIJ(0,86)=    0.73395       AIJ(0,99)=    0.78184
      AIJ(0,87)=    0.10270       AIJ(0,100)=   0.95241
C
C
C     RHS VALUES REQUESTED FOR EACH WEAPON CATEGORY.
C     ANY COEFFICIENTS MAY BE CHANGED IN THIS SECTION.  THE SESSION
C     IS TERMINATED WHEN THREE ZEROS ARE ENTERED.
C
 900  PRINT*,'INPUT ROW NUMBER,COLUMN NUMBER,COEFFICIENT:(I,J,COEFF)'
      PRINT*,'FOR EACH WEAPON RHS AND ANY OTHER COEFFICIENT TO BE'
      PRINT*,'CHANGED.  ENTER ONLY ONE SET AT A TIME. TO TERMINATE'
      PRINT*,'ENTER THREE ZEROS (0 0 0) '
      READ*, I,J,C
      IF(I.EQ.0.AND.J.EQ.0.AND.C.EQ.0) GO TO 902
      AIJ(I,J)=C
      GOTO 900
C
 902  PRINT*,'ENTER THE RHS WEAPON ALLOCATION VALUES'
       DO 46,I=11,15
        READ*,C
      AIJ(I,0)=C
  46  CONTINUE
C
 950  PRINT*,'THE FILE CONTAINING YOUR ARSENAL MODEL MATRIX'
      PRINT*,'IS CALLED-MN FOR ALL FORCES AND CV/CF FOR OPTIONS.'
C
C
C
C     ARSENAL MATRIX WRITEN TO FILE MTX.  ARSENAL MATRIX ALSO
C     COPIED FOR CV AND CF OPTIONS.
C
      OPEN(12,FILE='MN')
      REWIND 12
      DO 954, I=0,15
       DO 952, J=0,100
       IF(AIJ(I,J).NE.0.0)  WRITE(12,951) I,J,AIJ(I,J)
 951   FORMAT(2X,I3,2X,I3,2X,F10.5)
       CFIJ(I,J)=AIJ(I,J)
       CVIJ(I,J)=AIJ(I,J)
C
 952   CONTINUE
```

```
 954  CONTINUE
      CLOSE(12)
C
C     COUNTER-FORCE MATRIX: SET PK=0.0 FOR CV TARGETS IN OBJECTIVE
C
        OPEN(11,FILE='CF')
        REWIND 11
        DO 13, II=0,80,20
         DO 11, I=1,4
          CFIJ(0,I+II)= 0.0
  11     CONTINUE
         DO 12, I=17,20
          CFIJ(0,I+II)= 0.0
  12     CONTINUE
  13    CONTINUE
C
C
C
C     COUNTER-VALUE MATRIX: SET PK=0.0 FOR CF TARGETS IN OBJECTIVE
C
        OPEN(13,FILE='CV')
        REWIND 13
        DO 16,II=0,80,20
         DO 15,I=5,16
         CVIJ(0,I+II)= 0.0
 15      CONTINUE
 16     CONTINUE
C
C
C
C
C
C
       DO 21, I=0,15
         DO 20,J=0,100
           IF(CFIJ(I,J).NE.0.0) WRITE(11,29) I,J,CFIJ(I,J)
           IF(CVIJ(I,J).NE.0.0) WRITE(13,29) I,J,CVIJ(I,J)
  20     CONTINUE
  21   CONTINUE
  29  FORMAT(2X,I3,2X,I3,2X,F10.5)
C
      CLOSE(11)
      CLOSE(13)
C
      STOP
      END
```

Appendix B

This section contains the experimental designs used in the study. The designs are explained in Chapter III. The random data points are also contained in this appendix. The random data points were used to test the fit of the surface to a random set of points to the surface.

EXPERIMENTAL DESIGN POINTS

BIAS DESIGN (2 VARIABLES)

W1	W2	RESPONSE
113	188	255.789550
113	563	548.390880
338	188	446.673460
338	563	718.891660
66	375	368.951800
384	375	636.373700
225	640	686.068030
225	110	284.720310
225	225	381.244030

VARIANCE DESIGN (2 VARIABLES)

W1	W2	RESPONSE
000	750	574.755680
450	750	824.277880
000	000	0.000000
450	000	382.358800
000	375	312.936640
450	375	688.913240
225	000	191.243410
225	750	718.455880
225	375	503.584490

BIAS DESIGN (3 VARIABLES)

W1	W2	W3	RESPONSE
326	543	753	936.892380
326	543	287	883.645210
326	207	753	820.779310
326	207	287	669.880260
124	543	753	837.946050
124	543	287	738.305280
124	207	753	665.235910
124	207	287	499.332690
394	375	520	893.147910
56	375	520	651.915660
225	657	520	894.097370
225	93	520	604.202090
225	375	911	859.986050
225	375	129	610.025140
225	375	520	786.167230

VARIANCE DESIGN (3 VARIABLES)

W1	W2	W3	RESPONSE
450	750	520	1023.035450
450	000	520	723.572870
000	000	520	345.113760
000	750	520	797.826350
000	375	000	314.400160
000	375	1040	723.161960
450	375	1040	981.813950
450	375	000	695.335010
225	000	000	191.250000
225	750	000	787.683080
225	750	1040	965.302085
225	000	1040	669.632480
225	375	520	786.167230
224	374	519	784.606740
226	376	521	787.634740

RANDOM DATA POINTS

TWO VARIABLES

W1	W2	RESPONSE
74	614	551.81298
84	615	560.84159
329	263	500.29153
259	125	326.36002
137	630	615.51921
75	263	284.97425
435	152	498.35396
175	481	542.30104
142	573	579.57019
355	146	425.54636
18	250	225.89286
335	250	494.76924
41	7	40.8
190	93	240.53123
73	60	113.05

THREE VARIABLES

W1	W2	W3	RESPONSE
47	113	16	149.59712
100	349	265	575.78645
108	362	234	578.78938
189	698	64	766.14644
169	299	851	782.93449
351	52	114	439.02531
448	547	587	978.43160
433	689	56	917.37715
403	107	661	795.06902
384	276	450	821.86015
130	521	917	855.74153
286	307	501	785.47078
42	704	547	812.63913
46	458	910	781.90820
32	730	738	838.28725
230	96	539	617.68826
11	160	545	481.68624
5	406	347	570.93085
235	404	482	799.21689
32	732	351	785.572485
219	684	893	934.53033
244	439	233	746.9563
147	243	58	378.72357
132	202	911	707.60477
11	248	300	441.56219

Appendix C

```
*********************************************
* BIAS DESIGN ESTIMATION ( TWO VARIABLES ) *
*********************************************


        ASD COMPUTER CENTER
        WRIGHT-PATTERSON AFB,OHIO

        S P S S - - STATISTICAL PACKAGE FOR THE SOCIAL SCIENCES

        VERSION 9.0 (NOS) -- MARCH 06, 1984

        376500 CM MAXIMUM FIELD LENGTH REQUEST

                 RUN NAME       RESPONSE SURFACE FIT
                 VARIABLE LIST  W1,W2,AF
                 INPUT FORMAT   FREEFIELD
                 INPUT MEDIUM   CARD
                 N OF CASES     UNKNOWN
                 COMPUTE        W11=W1*W1
                 COMPUTE        W22=W2*W2
                 COMPUTE        W12=W1*W2
                 COMPUTE        AFL=LN(AF)
                 COMPUTE        W1L=LN(W1)
                 COMPUTE        W2L=LN(W2)
                 COMPUTE        W1L2L=LN(W1)*LN(W2)
                 COMPUTE        W12L=W1*LN(W2)
                 COMPUTE        W21L=W2*LN(W1)

  CPU TIME REQUIRED..      .051 SECONDS


                 REGRESSION     VARIABLES=AF,W1,W2,W12,W11,W22,
                                AFL,W1L,W2L,W1L2L,W12L,W21L/
                                REGRESSION = AF WITH W1,W2/
                                REGRESSION = AFL WITH W1L,W2L,W1L2L/
                                REGRESSION = AFL WITH W1L,W2L/
                                REGRESSION = AF WITH W1,W2,W12L,W21L/
                 STATISTICS     ALL
                 READ INPUT DATA
```

VARIABLE	MEAN	STANDARD DEV	CASES
AF	480.7893	173.2979	9
W1	225.2222	112.4653	9
W2	358.5556	194.0175	9
W12	80758.4444	62560.9798	9
W11	61968.1111	51623.6080	9
W22	162022.3333	148832.2058	9
AFL	6.1137	.3795	9
W1L	5.2766	.6015	9
W2L	5.7301	.6110	9
W1L2L	30.2172	4.6792	9
W12L	1290.5691	674.2853	9
W21L	1889.6317	1059.8697	9

CORRELATION COEFFICIENTS.

A VALUE OF 99.00000 IS PRINTED
IF A COEFFICIENT CANNOT BE COMPUTED.

	AF	W1	W2	W12	W11	W22	AFL	W1L	W2L	W1L2L	W12L
W1	.53360										
W2	.84472	.00019									
W12	.94504	.67472	.69864								
W11	.54738	.98150	.02127	.67695							
W22	.82662	.00021	.98561	.68859	-.00575						
AFL	.99075	.52688	.83340	.90573	.54571	.80579					
W1L	.49373	.97286	-.02241	.64061	.91244	.00955	.47910				
W2L	.83137	.00020	.97564	.68159	.05060	.92651	.83318	-.05680			
W1L2L	.95225	.73308	.66508	.96986	.72288	.65487	.93701	.71418	.65668		
W12L	.68963	.97675	.19929	.81351	.96895	.18928	.67577	.93936	.20424	.85360	
W21L	.93108	.20738	.97464	.83701	.21541	.96743	.90900	.19072	.94403	.80689	.39978

1RESPONSE SURFACE FIT 84/11/21. 16.44.31. PAGE 6

0* * * * * * * * * * * * * * * * * * M U L T I P L E R E G R E S S I O N * * * * * * * * * * * * * * * *
0DEPENDENT VARIABLE.. AF

MEAN RESPONSE 480.78927 STD. DEV. 173.29790

VARIABLE(S) ENTERED ON STEP NUMBER 1.. W1
W2

MULTIPLE R	.99906	ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE
R SQUARE	.99811	REGRESSION	2.	239804.02954	119902.01477
ADJUSTED R SQUARE	.99748	RESIDUAL	6.	453.25450	75.54242
STD DEVIATION	8.69151	COEFF OF VARIABILITY	1.8 PCT	F	SIGNIFICANCE
				1587.21445	.000

------------------- VARIABLES IN THE EQUATION -------------------

VARIABLE	B	STD ERROR B	F / SIGNIFICANCE	BETA / ELASTICITY
W1	.82197447	.27323221E-01	905.00895	.5334376
			.000	.38505
W2	.75442105	.15838338E-01	2268.8635	.8446200
			.000	.56262
(CONSTANT)	25.160493	8.8599901	8.0643953	
			.030	

ALL VARIABLES ARE IN THE EQUATION.

VARIANCE/COVARIANCE MATRIX

	W1	W2
W1	.00075	
W2	-.00000	.00025

FILE NONAME (CREATION DATE = 84/11/21.)

0* * * * * * * * * * * * * * * * * M U L T I P L E   R E G R E S S I O N * * * * * * * * * * * * * *

0DEPENDENT VARIABLE.. AFL

MEAN RESPONSE 6.11370 STD. DEV. .37946

VARIABLE(S) ENTERED ON STEP NUMBER 1.. W1L
W2L
W1L2L

MULTIPLE R	.99710	ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE
R SQUARE	.99422	REGRESSION	3.	1.14528	.38176
ADJUSTED R SQUARE	.99075	RESIDUAL	5.	.00666	.00133
STD DEVIATION	.03650	COEFF OF VARIABILITY	.6 PCT	F	SIGNIFICANCE
				286.52212	.000

------------------- VARIABLES IN THE EQUATION -------------------

VARIABLE	B	STD ERROR B	F / SIGNIFICANCE	BETA / ELASTICITY
W1L	1.8290258	.34358425	28.338259	2.8993632
			.003	1.57860
W2L	1.9025979	.31396508	36.722423	3.0632875
			.002	1.78324
W1L2L	-.25506796	.58471640E-01	19.029232	-3.1452610
			.007	-1.26068
(CONSTANT)	-6.7320874	1.8454840	13.306963	
			.015	

VARIANCE/COVARIANCE MATRIX OF THE UNNORMALIZED REGRESSION COEFFICIENTS.

W1L	.11805		
W2L	.10744	.09857	
W1L2L	-.02005	-.01832	.00342
	W1L	W2L	W1L2L

FILE NONAME (CREATION DATE = 84/11/21.)

0* * * * * * * * * * * * * * * * M U L T I P L E R E G R E S S I O N * * * * * * * * * * * * * * * *

0DEPENDENT VARIABLE.. AFL

MEAN RESPONSE 6.11370 STD. DEV. .37946

VARIABLE(S) ENTERED ON STEP NUMBER 1.. W1L
W2L

MULTIPLE R	.98601	ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE
R SQUARE	.97221	REGRESSION	2.	1.11992	.55996
ADJUSTED R SQUARE	.96294	RESIDUAL	6.	.03202	.00534
STD DEVIATION	.07305	COEFF OF VARIABILITY	1.2 PCT	F	SIGNIFICANCE
				104.93932	.000

-------------------- VARIABLES IN THE EQUATION --------------------

VARIABLE	B	STD ERROR B	F / SIGNIFICANCE	BETA / ELASTICITY
W1L	.33315945	.43004505E-01	60.017287	.5281228
			.000	.28754
W2L	.53611674	.42340503E-01	160.32701	.8631775
			.000	.50248
(CONSTANT)	1.2837147	.34234712	14.060594	
			.010	

ALL VARIABLES ARE IN THE EQUATION.

VARIANCE/COVARIANCE MATRIX OF THE UNNORMALIZED REGRESSION COEFFICIENTS.

W1L	.00185	
W2L	.00010	.00179
	W1L	W2L

FILE NONAME (CREATION DATE = 84/11/21.)

0* * * * * * * * * * * * * * * * M U L T I P L E R E G R E S S I O N * * * * * * * * * * * * * * * *

0DEPENDENT VARIABLE.. AF

MEAN RESPONSE 480.78927 STD. DEV. 173.29790

VARIABLE(S) ENTERED ON STEP NUMBER 1.. W1
W2
W21L
W12L

MULTIPLE R	.99981	ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE
R SQUARE	.99963	REGRESSION	4.	240168.16374	60042.04093
ADJUSTED R SQUARE	.99926	RESIDUAL	4.	89.12030	22.28007
STD DEVIATION	4.72018	COEFF OF VARIABILITY	1.0 PCT	F	SIGNIFICANCE
				2694.87615	.000

-------------------- VARIABLES IN THE EQUATION --------------------

VARIABLE	B	STD ERROR B	F / SIGNIFICANCE	BETA / ELASTICITY
W1	.52838286	.19150545	7.6126375	.3429051
			.051	.24752
W2	1.1537230	.10367612	123.83563	1.2916626
			.000	.86041
W21L	-.85208558E-01	.21158623E-01	16.217788	-.5211256
			.016	-.33489
W12L	.78547412E-01	.35353191E-01	4.9363567	.3056203
			.090	.21084
(CONSTANT)	7.7538571	11.089156	.48892087	
			.523	

ALL VARIABLES ARE IN THE EQUATION.

VARIANCE/COVARIANCE MATRIX OF THE UNNORMALIZED REGRESSION COEFFICIENTS.

	W1	W2	W12L	W21L
W1	.03667			
W2	-.00182	.01075		
W12L	-.00663	.00102	.00125	
W21L	.00120	-.00214	-.00035	.00045

STEPWISE ESTIMATION OF POLYNOMIAL

```
            REGRESSION     METHOD=STEPWISE/VARIABLES=AF,W1,W2,W12,W11,W22/
                           REGRESSION = AF WITH W1,W2,W12,W11,W22/
            STATISTICS     ALL
```

* *

VARIABLE(S) ENTERED ON STEP NUMBER 4.. W22

MULTIPLE R	.99990	ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE
R SQUARE	.99979	REGRESSION	4.	240207.40101	60051.85025
ADJUSTED R SQUARE	.99958	RESIDUAL	4.	49.88303	12.47076
STD DEVIATION	3.53140	COEFF OF VARIABILITY	.7 PCT	F	SIGNIFICANCE
				4815.41330	.000

------------------- VARIABLES IN THE EQUATION --------------------

VARIABLE	B	STD ERROR B	F / SIGNIFICANCE	BETA / ELASTICITY
W12	-.24194505E-03	.83706574E-04	8.3543822	-.0873428
			.045	-.04064
W2	.99273026	.42487624E-01	545.93085	1.1114215
			.000	.74034
W1	.91277073	.33314990E-01	750.66132	.5923617
			.000	.42758
W22	-.24311897E-03	.49632284E-04	23.994361	-.2087961
			.008	-.08193
(CONSTANT)	-21.806124	9.6085593	5.1503936	
			.086	

---------- VARIABLES NOT IN THE EQUATION -----------

VARIABLE	PARTIAL	TOLERANCE	F / SIGNIFICANCE
W11	.16713	.01118	.86208041E-01
			.788

0* * * * * * * * * * * * * * * * * * * M U L T I P L E   R E G R E S S I O N * * * * * * * * * * * * * * * *

0DEPENDENT VARIABLE..   AF

VARIABLE(S) ENTERED ON STEP NUMBER   5..   W11

MULTIPLE R	.99990	ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE
R SQUARE	.99980	REGRESSION	5.	240208.79440	48041.75888
ADJUSTED R SQUARE	.99946	RESIDUAL	3.	48.48963	16.16321
STD DEVIATION	4.02035	COEFF OF VARIABILITY	.8 PCT	F	SIGNIFICANCE
				2972.29069	.000

-------------------- VARIABLES IN THE EQUATION --------------------

VARIABLE	B	STD ERROR B	F / SIGNIFICANCE	BETA / ELASTICITY
W12	-.24192371E-03	.95296531E-04	6.4446997	-.0873351
			.085	-.04064
W2	.97352646	.81348377E-01	143.21804	1.0899217
			.001	.72602
W1	.87831110	.12334072	50.708855	.5699984
			.006	.41144
W22	-.21829319E-03	.10169538E-03	4.6076334	-.1874752
			.121	-.07356
W11	.76468259E-04	.26043978E-03	.86208041E-01	.0227791
			.788	.00986
(CONSTANT)	-15.922070	22.831374	.48633435	
			.536	

ALL VARIABLES ARE IN THE EQUATION.

VARIANCE/COVARIANCE MATRIX OF THE UNNORMALIZED REGRESSION COEFFICIENTS.

W1	.01521				
W2	.00844	.00662			
W12	-.00000	-.00000	.00000		
W11	-.00003	-.00002	.00000	.00000	
W22	-.00001	-.00001	.00000	.00000	.00000
	W1	W2	W12	W11	W22

```
1PAGE   2 BMDP    ESTIMATION OF CES PRODUCTION FUNCTION


 VARIABLES TO BE USED
      1 W1           2 W2           3 W3           4 AF
0INPUT FORMAT IS
 (F3.0,1X,F3.0,1X,F3.0,1X,F10.6)
0MAXIMUM LENGTH DATA RECORD IS   22 CHARACTERS.
0I N P U T   V A R I A B L E S . . . . .
   VARIABLE      RECORD   COLUMNS    FIELD  TYPE     VARIABLE      RECORD   COLUMNS    FIELD  TYPE
  INDEX   NAME     NO.  BEGIN  END   WIDTH          INDEX   NAME     NO.  BEGIN  END   WIDTH
  ----- --------  ----  -----  ----- -----  ----    ----- --------  ----  -----  ----- -----  ----
      1 W1           1      1     3     3    F          3 W3           1      9    11     3    F
      2 W2           1      5     7     3    F          4 AF           1     13    22  10.6    F
1PAGE   3 BMDP    ESTIMATION OF CES PRODUCTION FUNCTION


   REGRESSION TITLE
 ESTIMATION OF CES PRODUCTION FUNCTION

   REGRESSION NUMBER . . . . . . . . . . . . . . . .      0
   DEPENDENT VARIABLE. . . . . . . . . . . . . . . .    AF
   WEIGHTING VARIABLE. . . . . . . . . . . . . . . .
   NUMBER OF PARAMETERS. . . . . . . . . . . . . . .      4
   NUMBER OF CONSTRAINTS . . . . . . . . . . . . . .      0
   TOLERANCE FOR PIVOTING. . . . . . . . . . . . . . 1.0E-08
   TOLERANCE FOR CONVERGENCE . . . . . . . . . . . . 1.0E-05
   MAXIMUM NUMBER OF ITERATIONS. . . . . . . . . . .     50
   MAXIMUM NUMBER OF INCREMENT HALVINGS. . . . . . .      5


 PARAMETERS TO BE ESTIMATED

                    1 P1            2 P2            3 P3            4 P4
 MINIMUM      -.212676E+38    -.212676E+38    -.212676E+38    -.212676E+38
 MAXIMUM       .212676E+38     .212676E+38     .212676E+38     .212676E+38
 INITIAL         12.750000         .460000         .350000         .700000

   USING THE ABOVE SPECIFICATIONS THIS PROGRAM COULD USE UP TO  450 CASES.
 0BASED ON INPUT FORMAT SUPPLIED     1 RECORDS READ PER CASE.

 NUMBER OF CASES READ. . . . . . . . . . . . . .      15


   VARIABLE                       STANDARD
   NO. NAME           MEAN        DEVIATION      MINIMUM        MAXIMUM

    1  W1        225.000000     99.545395      56.000000     394.000000
    2  W2        375.000000    165.796777      93.000000     657.000000
    3  W3        520.000000    229.918308     129.000000     911.000000
    4  AF        756.770569    132.705988     499.332690     936.892380
```

CASE NO.	RESIDUAL	OBSERVED 3 AF	PREDICTED 3 AF	COOK DISTANCE	STD. DEV. PREDICTED	1 W1	2 W2
1	-2.895281	255.789550	258.684831	.021345	3.809592	113.000000	188.000000
2	3.716946	548.390880	544.673934	.081861	4.744735	113.000000	563.000000
3	.263276	446.673460	446.410184	.000317	4.467590	338.000000	188.000000
4	-6.558510	718.891660	725.450170	.305606	4.931611	338.000000	563.000000
5	4.990973	368.951800	363.960827	.152102	4.776147	66.000000	375.000000
6	12.691570	636.373700	623.682130	.745308	4.480298	384.000000	375.000000
7	-5.816733	686.068030	691.884763	.193911	4.709762	225.000000	640.000000
8	-7.968606	284.720310	292.688916	.211123	4.113054	225.000000	110.000000
9	-.027415	381.244030	381.271445	.000001	3.318322	225.000000	225.000000

```
******************************
**** THREE VARIABLE CASE ****
******************************
```

1PAGE 1

BMDPAR--DERIVATIVE-FREE NONLINEAR REGRESSION
BMDP STATISTICAL SOFTWARE, INC.
1964 WESTWOOD BLVD. SUITE 202
(213) 475-5700
PROGRAM REVISED APRIL 1982
MANUAL REVISED -- 1981

PROGRAM CONTROL INFORMATION

```
/PROBLEM     TITLE IS 'ESTIMATION OF CES PRODUCTION FUNCTION'.
/INPUT       VARIABLES ARE 4.
             FORMAT IS '(F3.0,1X,F3.0,1X,F3.0,1X,F10.6)'.
             UNIT IS 9.
/VARIABLE    NAMES ARE W1,W2,W3,AF.
/REGRESS     DEPENDENT IS AF.
             PARAMETERS ARE 4.
/PARAMETER   INITIAL ARE 12.75,.46,.35,.70.
/END
```

BMDP UNIT NO. 9 SPECIFIED IN THE INPUT PARAGRAPH
WILL REFER TO LOCAL FILE NAME BD5K3C FOR THIS PROBLEM.

```
NUMBER OF VARIABLES TO READ IN. . . . . . . . .        4
TOTAL NUMBER OF VARIABLES . . . . . . . . . . .        4
NUMBER OF CASES TO READ IN. . . . . . . . . . . TO END
INPUT UNIT NUMBER . . . . . . . . . . . . . . .        9
REWIND INPUT UNIT PRIOR TO READING. . DATA. . .      YES
```

VARIABLE NO.	NAME	MEAN	STANDARD DEVIATION	MINIMUM	MAXIMUM
1	W1	225.222222	112.465303	66.000000	384.000000
2	W2	358.555556	194.017468	110.000000	640.000000
3	AF	480.789269	173.297895	255.789550	718.891660

ITER. NO.	INCR. HALV.	RESIDUAL SUM OF SQUARES	PARAMETERS 1 P1	2 P2	3 P3

USER ROUTINE FUN LOADED FROM FILE LGO AT ADDRESS 077656B.

0	0	380069.453937	1.000000	.550000	1.000000
0	0	351591.813292	1.000000	.500000	1.000000
0	0	250095.842247	1.100000	.500000	1.000000
0	0	30051.998468	1.000000	.500000	1.100000
1	0	19915.686880	2.104949	.532411	.943083
2	0	8192.384342	2.247996	.521871	.937137
3	0	571.951114	2.383673	.521616	.935671
4	0	458.046663	2.370562	.520632	.939374
5	0	348.665033	2.337365	.520569	.940588
6	2	348.650220	2.336918	.520601	.940628
7	0	348.601419	2.338077	.520531	.940561
8	0	348.601283	2.338310	.520535	.940544
9	0	348.601278	2.338360	.520536	.940540
10	2	348.601278	2.338361	.520536	.940540
11	0	348.601278	2.338366	.520536	.940540
12	0	348.601278	2.338364	.520536	.940540

THE RESIDUAL SUM OF SQUARES (= 348.601) WAS SMALLEST WITH THE FOLLOWING PARAMETER VALUES

1 P1	2 P2	3 P3
2.33836	.520536	.940540

ESTIMATE OF ASYMPTOTIC CORRELATION MATRIX

		P1 1	P2 2	P3 3
P1	1	1.0000		
P2	2	-.2113	1.0000	
P3	3	-.9978	.2518	1.0000

THE ESTIMATED MEAN SQUARE ERROR IS 58.10

ESTIMATES OF ASYMPTOTIC STANDARD DEVIATIONS OF PARAMETER ESTIMATES WITH 6 DEGREES OF FREEDOM ARE

1 P1	2 P2	3 P3
.227410	8.644698E-03	1.683183E-02

```
BMDP UNIT NO.  9 SPECIFIED IN THE INPUT  PARAGRAPH
WILL REFER TO LOCAL FILE NAME BD4K2C  FOR THIS PROBLEM.

PROBLEM TITLE IS
ESTIMATION OF CES(P=-1) PRODUCTION FUNCTION

NUMBER OF VARIABLES TO READ IN. . . . . . . . .       3
TOTAL NUMBER OF VARIABLES . . . . . . . . . . .       3
NUMBER OF CASES TO READ IN. . . . . . . . . . . TO END
INPUT UNIT NUMBER . . . . . . . . . . . . . . .       9
REWIND INPUT UNIT PRIOR TO READING. . DATA. . .     YES
1PAGE   2 BMDP    ESTIMATION OF CES(P=-1) PRODUCTION FUNCTION


VARIABLES TO BE USED
     1 W1          2 W2          3 AF
0INPUT FORMAT IS
(F3.0,1X,F3.0,1X,F10.6)
0MAXIMUM LENGTH DATA RECORD IS   18 CHARACTERS.
0I N P U T   V A R I A B L E S . . . . .
   VARIABLE      RECORD   COLUMNS     FIELD  TYPE        VARIABLE     RECORD   COLUMNS     FIELD  TYPE
  INDEX   NAME      NO.  BEGIN  END   WIDTH              INDEX   NAME      NO.  BEGIN  END   WIDTH
  -----  --------  ----  -----  ----- -----  ----        -----  --------  ----  -----  ----- -----  ----
      1 W1           1      1     3     3     F              3 AF          1      9    18   10.6   F
      2 W2           1      5     7     3     F
1PAGE   3 BMDP    ESTIMATION OF CES(P=-1) PRODUCTION FUNCTION



  REGRESSION TITLE
ESTIMATION OF CES(P=-1) PRODUCTION FUNCTION

  REGRESSION NUMBER . . . . . . . . . . . . . . . .       0
  DEPENDENT VARIABLE. . . . . . . . . . . . . . . .    AF
  WEIGHTING VARIABLE. . . . . . . . . . . . . . . .
  NUMBER OF PARAMETERS. . . . . . . . . . . . . . .       3
  NUMBER OF CONSTRAINTS . . . . . . . . . . . . . .       0
  TOLERANCE FOR PIVOTING. . . . . . . . . . . . . . 1.0E-08
  TOLERANCE FOR CONVERGENCE . . . . . . . . . . . . 1.0E-05
  MAXIMUM NUMBER OF ITERATIONS. . . . . . . . . . .      50
  MAXIMUM NUMBER OF INCREMENT HALVINGS. . . . . .         5

PARAMETERS TO BE ESTIMATED

                   1 P1              2 P2              3 P3
MINIMUM      -.212676E+38      -.212676E+38      -.212676E+38
MAXIMUM       .212676E+38       .212676E+38       .212676E+38
INITIAL          1.000000           .500000          1.000000

USING THE ABOVE SPECIFICATIONS THIS PROGRAM COULD USE UP TO  560 CASES.
0BASED ON INPUT FORMAT SUPPLIED    1 RECORDS READ PER CASE.
NUMBER OF CASES READ. . . . . . . . . . . . . .       9
```

Appendix D

This section contains the results of the CES production function when estimated using a nonlinear estimation technique. The first portion are the results for the two variable case; the second section are the results of the three variable case.

```
1PAGE   1

 BMDPAR--DERIVATIVE-FREE NONLINEAR REGRESSION
 BMDP STATISTICAL SOFTWARE, INC.
 1964 WESTWOOD BLVD. SUITE 202
 (213) 475-5700
 PROGRAM REVISED APRIL    1982
 MANUAL REVISED -- 1981
 COPYRIGHT (C) 1982 REGENTS OF UNIVERSITY OF CALIFORNIA

 TO SEE REMARKS AND A SUMMARY OF NEW FEATURES FOR
 THIS PROGRAM, STATE NEWS. IN THE PRINT PARAGRAPH.


 THIS VERSION OF BMDP HAS BEEN CONVERTED FOR USE ON
 CDC 6000 AND CYBER SERIES COMPUTERS BY
 BMDP PROJECT, VOGELBACK COMPUTING CENTER
 NORTHWESTERN UNIVERSITY
 2129 SHERIDAN ROAD
 EVANSTON, ILLINOIS 60201
 (312) 492-3681

 RELEASED AUGUST 1983 FOR FTN5 COMPILERS

 EXECUTED ON 84/11/21.AT 18.59.49.

 PROGRAM CONTROL INFORMATION

 /PROBLEM       TITLE IS 'ESTIMATION OF CES(P=-1) PRODUCTION FUNCTION'.
 /INPUT         VARIABLES ARE 3.
                FORMAT IS '(F3.0,1X,F3.0,1X,F10.6)'.
                UNIT IS 9.
 /VARIABLE      NAMES ARE W1,W2,AF.
 /REGRESS       DEPENDENT IS AF.
                PARAMETERS ARE 3.
 /PARAMETER     INITIAL ARE 1,.5,1.
 /END
```

ALL VARIABLES ARE IN THE EQUATION.

VARIANCE/COVARIANCE MATRIX OF THE UNNORMALIZED REGRESSION COEFFICIENTS.

	W1	W2	W3	W12	W13	W23	W11	W22	W33
W1	.01891								
W2	.00221	.00681							
W3	.00160	.00096	.00354						
W12	-.00001	-.00000	-.00000	.00000					
W13	-.00001	-.00000	-.00000	-.00000	.00000				
W23	-.00000	-.00000	-.00000	-.00000	-.00000	.00000			
W11	-.00003	-.00000	-.00000	.00000	.00000	.00000	.00000		
W22	-.00000	-.00001	-.00000	.00000	.00000	.00000	.00000	.00000	
W33	-.00000	-.00000	-.00000	.00000	.00000	.00000	.00000	.00000	.00000

```
**************************************************
* VARIANCE DESIGN ESTIMATION ( THREE VARIABLES ) *
**************************************************

0* * * * * * * * * * * * * * * * M U L T I P L E   R E G R E S S I O N * * * * * * * * * * * * * * *
0DEPENDENT VARIABLE..    AF

 MEAN RESPONSE      705.10239      STD. DEV.     243.30773

 VARIABLE(S) ENTERED ON STEP NUMBER   1..   W1
                                            W33
                                            W2
                                            W23
                                            W13
                                            W12
                                            W22
                                            W11
                                            W3

 MULTIPLE R            .99831        ANALYSIS OF VARIANCE    DF      SUM OF SQUARES        MEAN SQUARE
 R SQUARE              .99662        REGRESSION              9.        825979.88581        91775.54287
 ADJUSTED R SQUARE     .99054        RESIDUAL                5.          2801.20330          560.24066
 STD DEVIATION       23.66940        COEFF OF VARIABILITY    3.4 PCT                   F     SIGNIFICANCE
                                                                               163.81450         .000


 -------------------- VARIABLES IN THE EQUATION --------------------

 VARIABLE           B          STD ERROR B          F           BETA
                                               ------------  ----------
                                               SIGNIFICANCE  ELASTICITY

 W1           1.1681321       .13749718        72.176595     .8165836
                                                    .000       .37275
 W33        -.32614660E-03    .45554629E-04    51.257883    -.5790773
                                                    .001      -.19178
 W2           1.0879986       .82498084E-01   173.92778     1.2676083
                                                    .000       .57864
 W23        -.38559420E-03    .60690778E-04    40.365987    -.3693692
                                                    .001      -.10664
 W13        -.26128816E-03    .10115130E-03    6.6726226    -.1501764
                                                    .049      -.04336
 W12        -.45407410E-03    .14026313E-03    10.480124    -.1882074
                                                    .023      -.05434
 W22        -.31630721E-03    .87594558E-04    13.039590    -.2920712
                                                    .015      -.09673
 W11        -.38063888E-03    .243319160E-03   2.4472237    -.1265306
                                                    .179      -.04190
 W3           .86739792       .59493754E-01   212.56610     1.4013512
                                                    .000       .63969
 (CONSTANT)  -39.723798      27.964356         2.0178639
                                                    .215
```

-------------------- VARIABLES IN THE EQUATION --------------------

VARIABLE	B	STD ERROR B	F / SIGNIFICANCE	BETA / ELASTICITY
W12	-.60263960E-03	.32447497E-03	3.4494744	-.2466326
			.122	-.06719
W3	.86369212	.14355786	36.196280	1.4963803
			.002	.59347
W2	1.2460058	.19905066	39.184370	1.5567025
			.002	.61743
W1	1.2405823	.33206385	13.957510	.9305854
			.013	.36884
W23	-.52343376E-03	.14065224E-03	13.849374	-.4948856
			.014	-.13488
W33	-.30201080E-03	.11669216E-03	6.6982537	-.5555488
			.049	-.12760
W22	-.40276597E-03	.22434771E-03	3.2230146	-.3852893
			.133	-.08850
W13	-.32612592E-03	.23395620E-03	1.9431284	-.1850829
			.222	-.05042
W11	-.30196882E-03	.62425204E-03	.23399376	-.1040510
			.649	-.02389
(CONSTANT)	-66.041758	83.688299	.62274143	
			.466	

ALL VARIABLES ARE IN THE EQUATION.

VARIANCE/COVARIANCE MATRIX OF THE UNNORMALIZED REGRESSION COEFFICIENTS.

	W1	W2	W3	W12	W13	W23	W11	W22	W33
W1	.11027								
W2	.03813	.03962							
W3	.02750	.01650	.02061						
W12	-.00004	-.00002	-.00000	.00000					
W13	-.00003	-.00000	-.00001	-.00000	.00000				
W23	-.00000	-.00001	-.00001	-.00000	-.00000	.00000			
W11	-.00018	-.00006	-.00005	.00000	.00000	.00000	.00000		
W22	-.00004	-.00004	-.00002	.00000	.00000	.00000	.00000	.00000	
W33	-.00002	-.00001	-.00001	.00000	.00000	.00000	.00000	.00000	.00000

------------------- VARIABLES IN THE EQUATION -------------------

VARIABLE	B	STD ERROR B	F / SIGNIFICANCE	BETA / ELASTICITY
W1	.78473263	.13527292	33.652838	.5688254
			.004	.37867
W2	.74158196	.81216389E-01	83.374062	.8959116
			.001	.59641
W12L	.22738666E-01	.33438221E-01	.46242749	.1036654
			.534	.04589
W21L	-.26177781E-01	.21905433E-01	1.4281111	-.1827965
			.298	-.08088
(CONSTANT)	27.938445	36.914071	.57282319	
			.491	

ALL VARIABLES ARE IN THE EQUATION.

VARIANCE/COVARIANCE MATRIX OF THE UNNORMALIZED REGRESSION COEFFICIENTS.

W1	.01830			
W2	.00597	.00660		
W12L	-.00221	-.00036	.00112	
W21L	-.00041	-.00088	-.00049	.00048
	W1	W2	W12L	W21L

```
**********************************************
* BIAS DESIGN ESTIMATION ( THREE VARIABLES ) *
**********************************************
```

0* * * * * * * * * * * * * * * * * M U L T I P L E   R E G R E S S I O N * * * * * * * * * * * * * * * *

0DEPENDENT VARIABLE..   AF

VARIABLE(S) ENTERED ON STEP NUMBER   9..   W11

MULTIPLE R	.99754	ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE
R SQUARE	.99508	REGRESSION	9.	245339.80380	27259.97820
ADJUSTED R SQUARE	.98623	RESIDUAL	5.	1212.50548	242.50110
STD DEVIATION	15.57245	COEFF OF VARIABILITY	2.1 PCT	F	SIGNIFICANCE
				112.41177	.000

------------------- VARIABLES IN THE EQUATION -------------------

VARIABLE	B	STD ERROR B	F / SIGNIFICANCE	BETA / ELASTICITY
W1L	.37923771	.17176596	4.8747104	.5213492
			.069	.26571
W2L	.41957953	.15791222	7.0598769	.6274121
			.038	.32004
(CONSTANT)	2.2713777	1.0448554	4.7257021	
			.073	

ALL VARIABLES ARE IN THE EQUATION.

VARIANCE/COVARIANCE MATRIX OF THE UNNORMALIZED REGRESSION COEFFICIENTS.

	W1L	W2L
W1L	.02950	
W2L	-.00000	.02494

1RESPONSE SURFACE FIT 84/11/21. 16.45.12. PAGE 13

FILE NONAME (CREATION DATE = 84/11/21.)

0* * * * * * * * * * * * * * * * M U L T I P L E   R E G R E S S I O N * * * * * * * * * * * * * * * *

0DEPENDENT VARIABLE.. AF

MEAN RESPONSE 466.28067 STD. DEV. 268.81649

VARIABLE(S) ENTERED ON STEP NUMBER 1.. W1
W2
W12L
W21L

MULTIPLE R	.99265	ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE
R SQUARE	.98535	REGRESSION	4.	569630.97839	142407.74460
ADJUSTED R SQUARE	.97071	RESIDUAL	4.	8467.45393	2116.86348
STD DEVIATION	46.00938	COEFF OF VARIABILITY	9.9 PCT	F	SIGNIFICANCE
				67.27299	.001

MULTIPLE R	.99915	ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE
R SQUARE	.99829	REGRESSION	3.	35.46401	11.82134
ADJUSTED R SQUARE	.99727	RESIDUAL	5.	.06072	.01214
STD DEVIATION	.11020	COEFF OF VARIABILITY	2.0 PCT	F	SIGNIFICANCE
				973.44624	.000

-------------------- VARIABLES IN THE EQUATION --------------------

VARIABLE	B	STD ERROR B	F / SIGNIFICANCE	BETA / ELASTICITY
W1L	.97003215	.23224050E-01	1744.6019	1.3335316
			.000	.67966
W2L	.96226797	.21339018E-01	2033.4955	1.4389132
			.000	.73398
W1L2L	-.14125954	.45270158E-02	973.66943	-1.2849180
			.000	-.41394
(CONSTANT)	.16738532E-02	.10947133	.23379434E-03	
			.988	

ALL VARIABLES ARE IN THE EQUATION.

VARIANCE/COVARIANCE MATRIX OF THE UNNORMALIZED REGRESSION COEFFICIENTS.

W1L	.00054		
W2L	.00033	.00046	
W1L2L	-.00009	-.00008	.00002
	W1L	W2L	W1L2L

FILE NONAME (CREATION DATE = 04/11/21.)

* * * * * * 08 * M U L T I P L E   R E G R E S S I O N * * * * * * * * * * * * * * * * *

ODEPENDENT VARIABLE.. AFL

MEAN RESPONSE 5.48315 STD. DEV. 2.10727

VARIABLE(S) ENTERED ON STEP NUMBER 1.. W1L
W2L

MULTIPLE R	.81575	ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE
R SQUARE	.66545	REGRESSION	2.	23.63996	11.81998
ADJUSTED R SQUARE	.55393	RESIDUAL	6.	11.88477	1.98079
STD DEVIATION	1.40741	COEFF OF VARIABILITY	25.7 PCT	F	SIGNIFICANCE
				5.96729	.037

```
1RESPONSE SURFACE FIT                                        84/11/21.    16.45.12.    PAGE   7
 FILE   NONAME   (CREATION DATE =  84/11/21.)
0* * * * * * * * * * * * * * * * *  M U L T I P L E   R E G R E S S I O N  * * * * * * * * * * * * * * *
0DEPENDENT VARIABLE..    AF

 MEAN RESPONSE       466.28067      STD. DEV.     268.81649

 VARIABLE(S) ENTERED ON STEP NUMBER   1..   W1
                                            W2


 MULTIPLE R           .98997       ANALYSIS OF VARIANCE    DF      SUM OF SQUARES        MEAN SQUARE
 R SQUARE             .98004       REGRESSION               2.       566560.78681       283280.39340
 ADJUSTED R SQUARE    .97339       RESIDUAL                 6.        11537.64551         1922.94092
 STD DEVIATION      43.85135       COEFF OF VARIABILITY     9.4 PCT               F     SIGNIFICANCE
                                                                            147.31622         .000


 -------------------- VARIABLES IN THE EQUATION --------------------

 VARIABLE           B          STD ERROR B          F           BETA
                                               ------------  ----------
                                               SIGNIFICANCE  ELASTICITY

 W1            .74656119     .79565506E-01     88.040228      .5411562
                                                    .000        .36025
 W2            .68617210     .47739303E-01     206.59222      .8289705
                                                    .000        .55184
 (CONSTANT)    40.989864     29.234233         1.9659367
                                                    .210


 ALL VARIABLES ARE IN THE EQUATION.

 VARIANCE/COVARIANCE MATRIX OF THE UNNORMALIZED REGRESSION COEFFICIENTS.


 W1            .00633
 W2                 0     .00228

               W1         W2


 FILE   NONAME   (CREATION DATE =  84/11/21.)
0* * * * * * * * * * * * * * * * *  M U L T I P L E   R E G R E S S I O N  * * * * * * * * * * * * * * *
0DEPENDENT VARIABLE..    AFL

 MEAN RESPONSE        5.48315      STD. DEV.      2.10727

 VARIABLE(S) ENTERED ON STEP NUMBER   1..   W1L
                                            W2L
                                            W1L2L
```

FILE NONAME (CREATION DATE = 84/11/21.)

0* * * * * * * * * * * * * * * * * M U L T I P L E R E G R E S S I O N * * * * * * * * * * * * * * *

0DEPENDENT VARIABLE.. AF

MEAN RESPONSE 466.28067 STD. DEV. 268.81649

VARIABLE(S) ENTERED ON STEP NUMBER 1.. W1
W2
W12
W11
W22

MULTIPLE R	.99884	ANALYSIS OF VARIANCE	DF	SUM OF SQUARES	MEAN SQUARE
R SQUARE	.99768	REGRESSION	5.	576757.42575	115351.48515
ADJUSTED R SQUARE	.99381	RESIDUAL	3.	1341.00657	447.00219
STD DEVIATION	21.14243	COEFF OF VARIABILITY	4.5 PCT	F	SIGNIFICANCE
				258.05575	.000

-------------------- VARIABLES IN THE EQUATION --------------------

VARIABLE	B	STD ERROR B	F SIGNIFICANCE	BETA ELASTICITY
W1	.95834307	.14607661	43.040831 .007	.6946695 .46244
W2	1.0589761	.87645966E-01	145.98508 .001	1.2793582 .85167
W12	-.39358993E-03	.12528845E-03	9.8688344 .052	-.1747096 -.07122
W11	-.14263480E-03	.29530771E-03	.23329284 .662	-.0484257 -.02581
W22	-.37899507E-03	.10631078E-03	12.709034 .038	-.3574225 -.19050
(CONSTANT)	-12.391642	18.975908	.42643475 .560	

VARIANCE/COVARIANCE MATRIX OF THE UNNORMALIZED REGRESSION COEFFICIENTS.

	W1	W2	W12	W11	W22
W1	.02134				
W2	.00132	.00768			
W12	-.00001	-.00000	.00000		
W11	-.00004	0	0	.00000	
W22	0	-.00001	0	0	.00000

```
**************************************************
* VARIANCE DESIGN ESTIMATION ( TWO VARIABLES ) *
**************************************************
```

1RESPONSE SURFACE FIT   84/11/21.   16.45.12.   PAGE   4

FILE   NONAME   (CREATION DATE = 84/11/21.)

0* * * * * * * * * * * * * * * * * * M U L T I P L E   R E G R E S S I O N * * * * * * * * * * * * * * * *

VARIABLE	MEAN	STANDARD DEV	CASES
AF	466.2807	268.8165	9
W1	225.0000	194.8557	9
W2	375.0000	324.7595	9
W12	84375.0000	119324.2693	9
W11	84375.0000	91265.5167	9
W22	234375.0000	253515.3241	9
AFL	5.4831	2.1073	9
W1L	3.8418	2.8969	9
W2L	4.1823	3.1511	9
W1L2L	16.0676	19.1681	9
W12L	941.0249	1225.5323	9
W21L	1440.6685	1877.1157	9

CORRELATION COEFFICIENTS.

A VALUE OF 99.00000 IS PRINTED
IF A COEFFICIENT CANNOT BE COMPUTED.

	AF	W1	W2	W12	W11	W22	AFL	W1L	W2L	W1L2L	W12L
W1	.54116										
W2	.82897	0									
W12	.79535	.61237	.61237								
W11	.51620	.96077	0	.58835							
W22	.76896	0	.96077	.58835	0						
AFL	.79739	.48604	.57848	.41759	.41423	.48822					
W1L	.49964	.91317	.00000	.55920	.76430	.00000	.52135				
W2L	.79529	.00000	.90971	.55708	0	.75886	.62741	.00000			
W1L2L	.79520	.57720	.57454	.89179	.48311	.47926	.46675	.63209	.63156		
W12L	.79135	.66498	.52629	.94435	.63889	.43901	.43990	.60724	.57852	.94287	
W21L	.80521	.52848	.66467	.94640	.44233	.63859	.44228	.57873	.60466	.94070	.88665

ITER. NO.	INCR. HALV.	RESIDUAL SUM OF SQUARES	PARAMETERS 1 P1	2 P2	3 P3	4 P4
		USER ROUTINE FUN LOADED FROM FILE LGO AT ADDRESS 077656B.				
0	0	2012146.071836	12.750000	.460000	.350000	.770000
0	0	58356.798237	14.025000	.460000	.350000	.700000
0	0	32425.418843	12.750000	.506000	.350000	.700000
0	0	19025.495991	12.750000	.460000	.385000	.700000
0	0	11010.666724	12.750000	.460000	.350000	.700000
1	0	8479.300134	16.003967	.462151	.352606	.668145
2	0	6112.735411	16.896061	.462232	.352563	.655157
3	0	5922.102643	17.012997	.462337	.352620	.654497
4	5	230410.308065	14.991034	.461001	.352992	.646706
5	0	8533.044444	16.766211	.461002	.352067	.654226
6	0	5972.258448	16.765768	.464224	.350912	.658003
7	0	5880.106121	17.001340	.461848	.352814	.655003
8	0	5879.265444	17.004706	.461857	.352801	.654919
9	0	5879.264358	17.004038	.461852	.352805	.654923
10	0	5879.264165	17.003685	.461852	.352810	.654927
11	0	5879.264128	17.003843	.461854	.352812	.654926
12	0	5879.264124	17.003584	.461856	.352811	.654928
13	0	5879.264124	17.003645	.461855	.352811	.654928

THE RESIDUAL SUM OF SQUARES (= 5879.26) WAS SMALLEST WITH THE FOLLOWING PARAMETER VALUES

1 P1	2 P2	3 P3	4 P4
17.0036	.461855	.352811	.654928

ESTIMATE OF ASYMPTOTIC CORRELATION MATRIX

		P1 1	P2 2	P3 3	P4 4
P1	1	1.0000			
P2	2	-.0021	1.0000		
P3	3	.0973	-.7663	1.0000	
P4	4	-.9975	.0556	-.1251	1.0000

THE ESTIMATED MEAN SQUARE ERROR IS 534.5

ESTIMATES OF ASYMPTOTIC STANDARD DEVIATIONS OF PARAMETER ESTIMATES WITH 11 DEGREES OF FREEDOM ARE

1 P1	2 P2	3 P3	4 P4
3.27720	2.604367E-02	2.232392E-02	3.283604E-02

CASE NO.	RESIDUAL	OBSERVED 4 AF	PREDICTED 4 AF	COOK DISTANCE	STD. DEV. PREDICTED	1 W1	2 W2	3 W3
1	-34.881964	936.892380	971.774344	.355153	12.727619	326.0	543.0	753.0
2	29.828829	883.645210	853.816381	.201149	11.849887	326.0	543.0	287.0
3	13.143130	820.779310	807.636180	.040014	11.934051	326.0	207.0	753.0
4	-6.158348	669.880260	676.038608	.011607	12.888390	326.0	207.0	287.0
5	-6.039194	837.946050	843.985244	.008278	11.863760	124.0	543.0	753.0
6	22.487536	738.305280	715.817744	.148631	12.751218	124.0	543.0	287.0
7	.329927	665.235910	664.905983	.000034	12.919276	124.0	207.0	753.0
8	-17.141728	499.332690	516.474418	.062566	11.642453	124.0	207.0	287.0
9	17.808835	893.147910	875.339075	.065858	11.555498	394.0	375.0	520.0
10	11.974304	651.915660	639.941356	.041218	12.675628	56.0	375.0	520.0
11	-10.908281	894.097370	905.005651	.025402	11.651402	225.0	657.0	520.0
12	.094793	604.202090	604.107297	.000003	12.640563	225.0	93.0	520.0
13	-7.533425	859.986050	867.519475	.011779	11.553742	225.0	375.0	911.0
14	-39.082710	610.025140	649.107850	.441878	12.697190	225.0	375.0	129.0
15	23.711366	786.167230	762.455864	.020835	6.060161	225.0	375.0	520.0

NUMBER OF INTEGER WORDS OF STORAGE USED IN PRECEDING PROBLEM 671
CPU TIME USED .821 SECONDS

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This thesis represents a methodology for the identification of cost effective strategic force mixes. The methodology makes use of Response Surface Methodology (RSM), economic production functions, economic theory, deterministic models, and Lagrangian techniques to identify cost effective choices. The methodology fits economic production functions to the response surface of a nuclear exchange model (a linear programming problem) using RSM. It then maximizes these economic production functions subject to a cost constraint using the Lagrangian technique. The use of economic production functions in this manner gives economic insight into the problem and results in the development of some simple buy decision rules for determination of cost effective force mixes. The classical use of polynomial models does not provide the same degree of information as the economic production functions, and information gained from the polynomial requires much more work.

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